A Newly Proposed Methodology and Paradigm Shift Regarding to Flood Control as Disaster Reduction under the Uncertainty of Global Warming

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Recent Heavy Rainfall Disaster In Japan

Introduction
Serious flood disasters in Japan

The 2015 Kanto-Tohoku heavy rain disaster in Japan

Introduction

Recent serious flood disasters in Japan

July 2018 “Nishi Nihon Heavy Rain”

· Recordable heavy rain occurred in various parts of western Japan due to typhoon and baiu front.

· Floods of rivers and sediment disasters occurred in many areas, mainly in western Japan.

July 2017 “Northern Kyushu Heavy Rain”

· By influence of typhoon and Baiu front, flooding of rivers and large-scale landslides occurred.

· Damage caused by driftwood flowing into rivers was remarkable.

August 2016 “Hokkaido Heavy Rain”

· Recordable heavy rain over Hokkaido due to the landing and approach of the four typhoons.

· Unprecedented wide area damage (flooding, outflow of pier, agricultural damage).

September 2015 “Kanto-Tohoku Heavy Rain”

· Recordable heavy rainfall occurred in various places in the Tohoku region from the Kita Kanto region.

· Rainfall precipitation concentrated in the Kinugawa river system, resulting breaking of levee.

Introduction
Recent serious flood disasters in Japan

July 2018 “Nishi Nihon Heavy Rain”
- The number of dead：220
- The number of flooded houses：More than 34,200

July 2017 “Northern Kyushu Heavy Rain”
- The number of dead：37
- The number of flooded houses：More than 2,100

August 2016 “Hokkaido Heavy Rain”
- Damaged area：40,258 ha (3.5% of arable land area in Hokkaido)
- Total damage amount：3 billion USD (the highest amount ever recorded in Hokkaido)

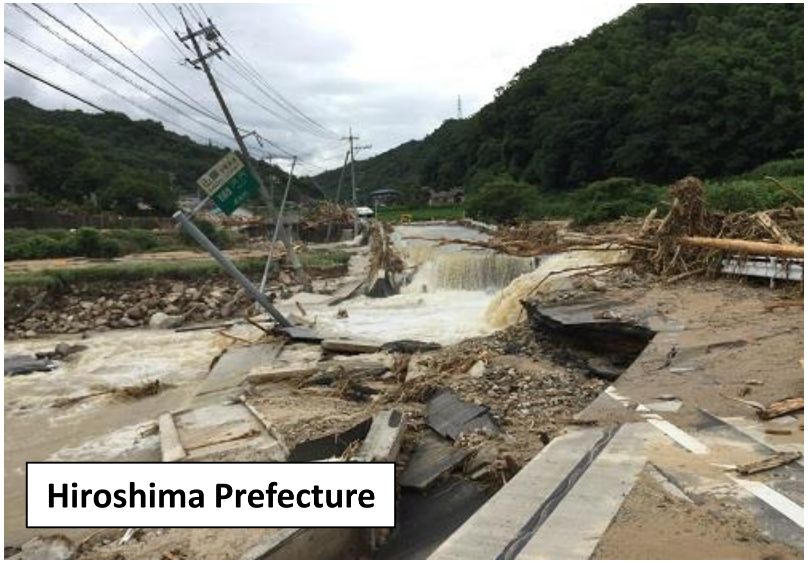
※1 USD = 100 yen

September 2015 “Kanto-Tohoku Heavy Rain”
- The number of dead：8
- Flooded house :More than 12,000

July 2018 “Nishi-nihon Heavy Rain”

(Source) 道路構造物ジャーナル NET

(Source)ふるさとチョイス



(Source) ふるさとチョイス

(Source)ふるさとチョイス

July 2017 "Northern Kyushu Heavy Rain" (As of 2018/09/19 12:00)

Photographed on 21st August 2017
A large amount of driftwood is scattered upstream of Myoukengawa river.

Photographed date unknown
A driftwood group approaching the private house in Turukawachi district along with the muddy stream

Photographed date unknown
Immediately after a disaster. You can see a vehicle drifted by driftwood.

Photographed date unknown
In Asakura city, near the Yamada intersection

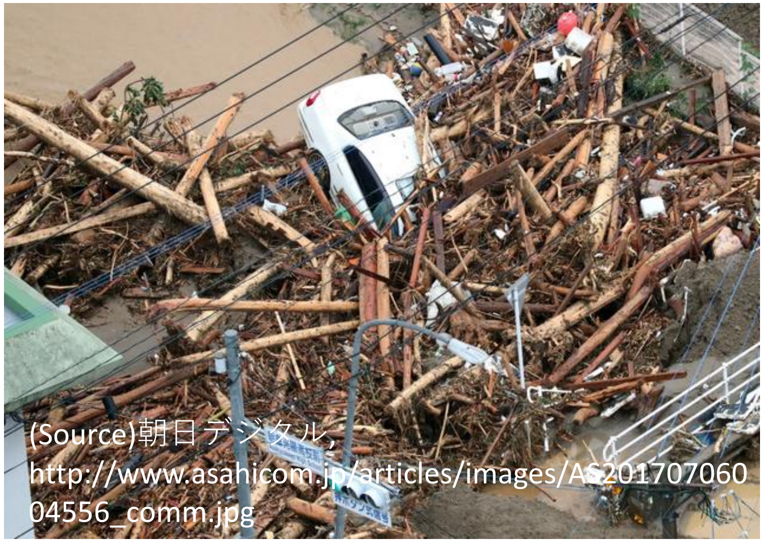
July 2017 “Northern Kyushu Heavy Rain” (As of 2018/09/19 12:00)

(Source)NHK HP, http://blog.hitachi-

(Source)時事ドットコム, https://www.jiji.com/jc/d4?p=tyh017-jpp024418481&d=d4_aum

July 6 afternoon, Asakura city, Fukuoka prefecture
River filled with a large amount of driftwood.

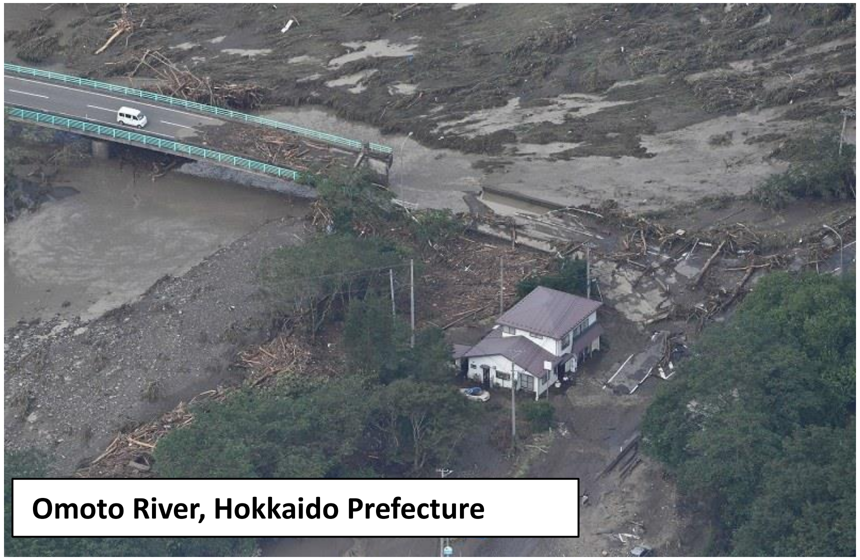
(Source)時事ドットコム, https://www.jiji.com/jc/d4?p=tyh017-jpp024418545&d=d4_aum



(Source)朝日デジタル, http://www.asahicom.jp/articles/images/AS20170706004556_comm.jpg

July 6 afternoon, Asakura city, Fukuoka prefecture Massive driftwood and private houses due to heavy rain.

August 2016 “Hokkaido Heavy Rain”



Omoto River, Hokkaido Prefecture

(Source)毎日新聞

Omoto River, Hokkaido Prefecture

(Source)毎日新聞

Sorachi River, Hokkaido Prefecture

(Source)国土交通省「平成２８年台風第１０号による出水状況について」

Sorachi River, Hokkaido Prefecture

(Source)国土交通省「平成２８年台風第１０号による出水状況について」

September 2015 “Kanto-Tohoku Heavy Rain”


(Source)Signal


(Source) 国土交通省関東地方整備局


(Source)ピースボート災害ボランティアセンター

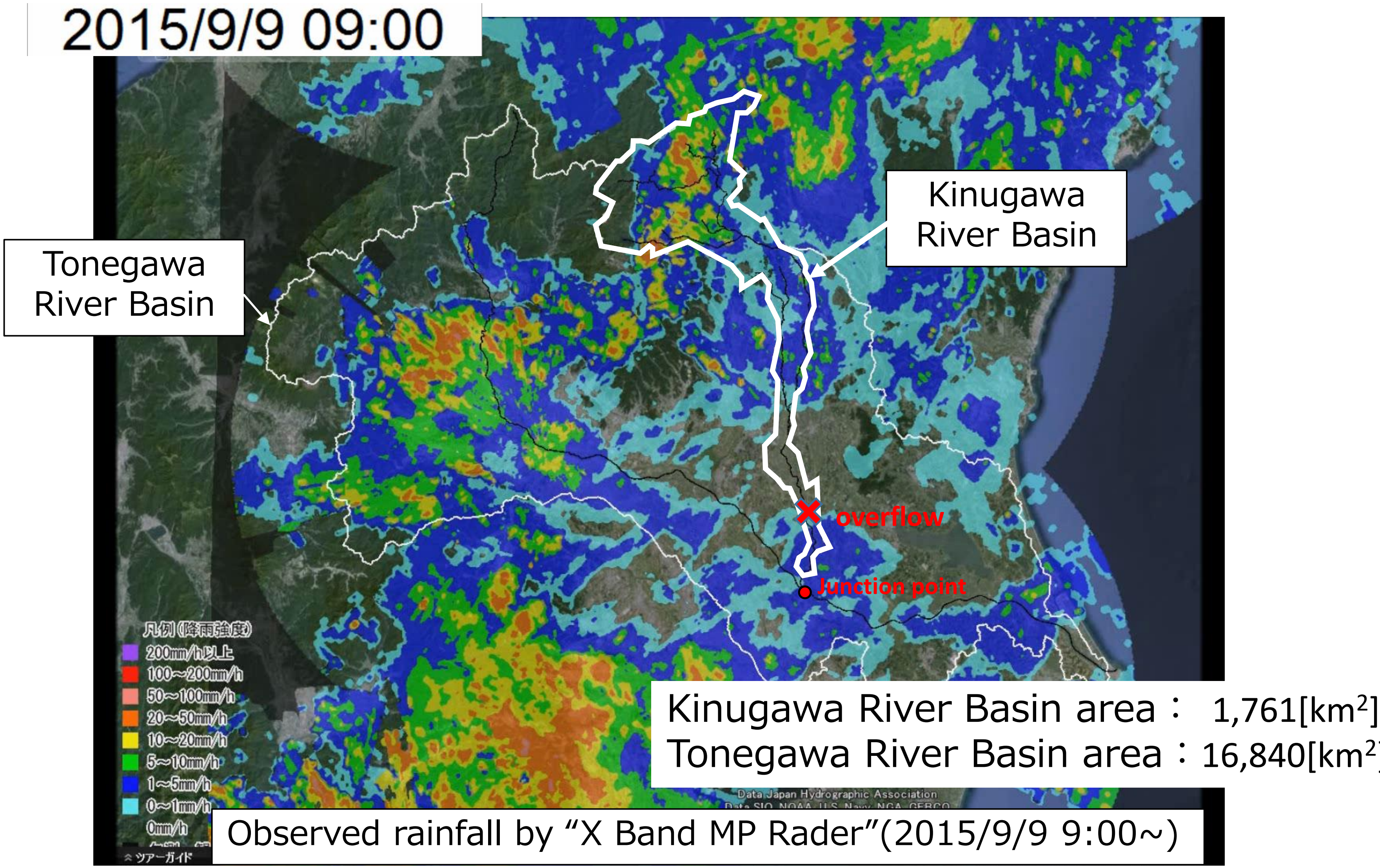
(Source) ほっとメール@ひたち

Chapter 2

Relation between evacuation information and situation of inundation at flooding

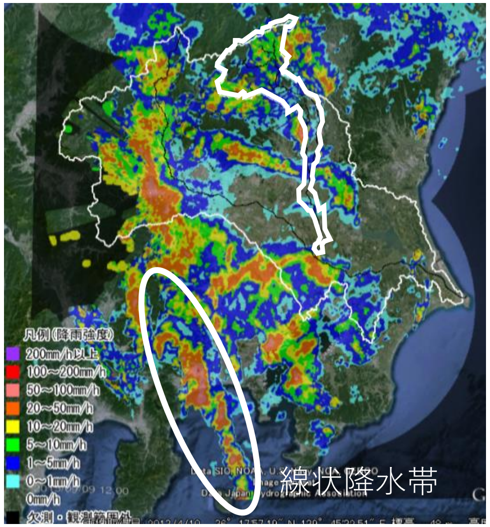
How to deal with serious flood disaster?

Heavy rain in the Kanto and Tohoku regions, September 2015

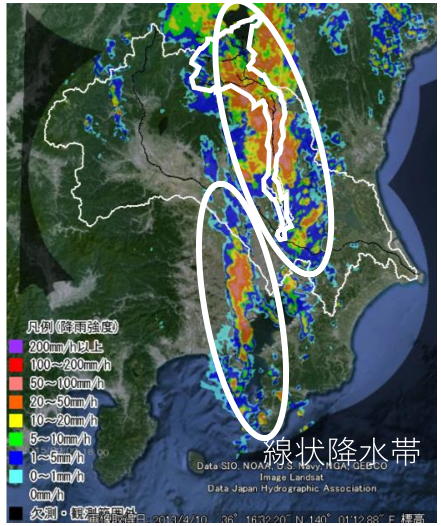


How to deal with serious flood disaster?

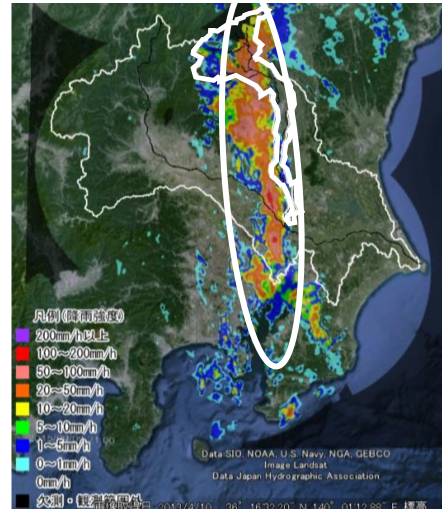
Heavy rain in the Kanto and Tohoku regions, September 2015



2015/9/9 12:00



18:00



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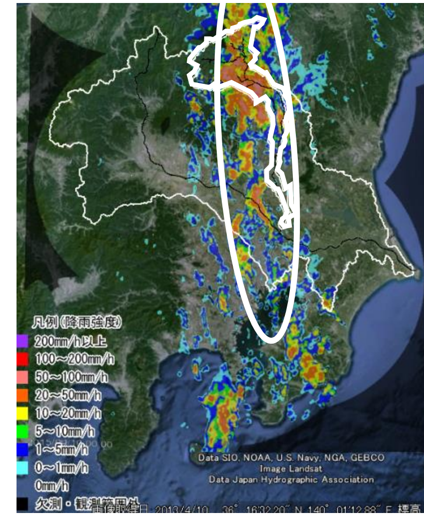

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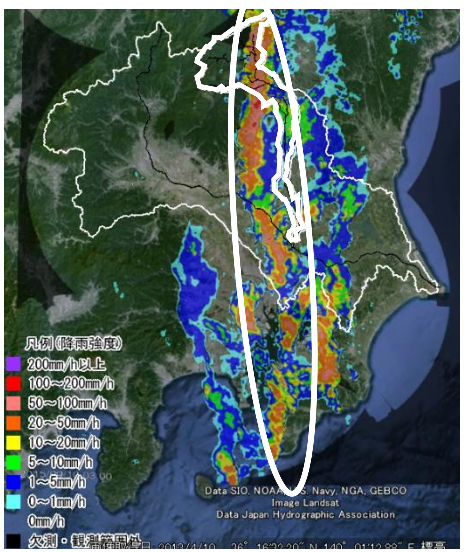
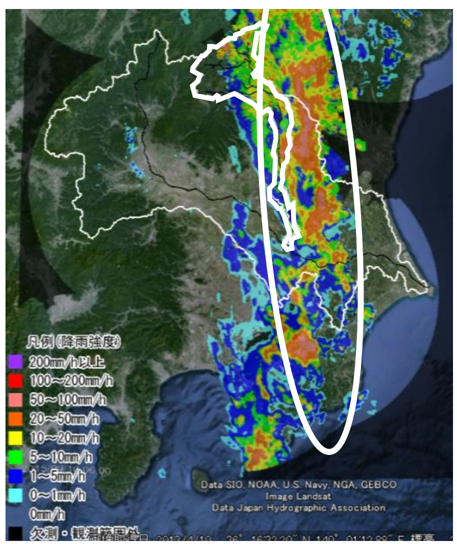
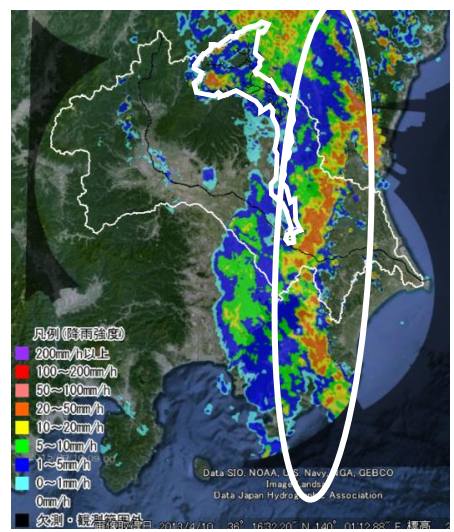


2015/9/10 3:00



6:00



9:00

At Kinugawa river Basin (Especially upstream area), heavy rainfall continued from 12a.m 9 Sep. to 10a.m 10 Sep. because of Band-Shaped Precipitation System.(Precipitation:50 ~100[mm/h])

Heavy rain in the Kanto and Tohoku regions, Sep.2015
Outline of Kinugawa River Basin

鬼怒川・小貝川の分離

Until early days of Edo Period, Kokai river jointed to Kinugawa river.
And Kinugawa river jointed Hitachi river(Tonegawa river).
In 1629, Kinugawa river and Kokai river are separated.

鬼怒川の河道変遷

年	内　容
神護景雲２年（768 年）	鬼怒川流路開削。大渡戸から桐ケ瀬（現下妻市）に至る流路が開削される。〔毛野川（鬼怒川）を掘って新しい水路をつくって洪水を防いで田畑や用水路を守るという目的があったという記録がのこる『続日本紀』〕
承平年間（931～938 年）	糸繰川を通じて小貝川を合わせていた鬼怒川は、別れて南流し、糸繰川部分は旧河道となった。下流の谷和原村寺畑地先（現つくばみらい市）で再び鬼怒川と合流していた。
寛永６年（1629 年）	大木の開削。大木台地（守谷市）を掘削して常陸川（現利根川）につなげた。
寛永７年（1630 年）	鬼怒川と小貝川を分離。鬼怒川を谷和原村寺畑地先で締め切り、小貝川と分離した。（谷和原の開発と鬼怒川舟運の整備が目的とされる。）

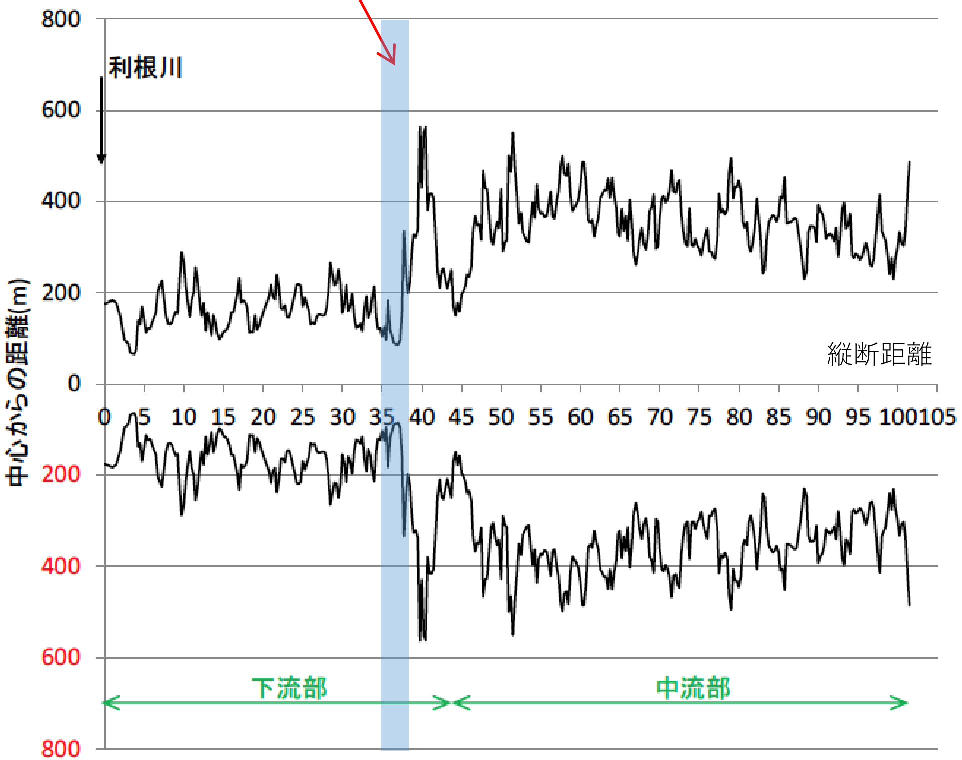
「明治以前日本土木史」他による

Quoted from「第1回鬼怒川・小貝川有識者会議」
Kanto Regional Development Bureau, Ministry of Land, Infrastructure, Transportation and Tourism

Heavy rain in the Kanto and Tohoku regions, Sep.2015

Outline of Kinugawa River Basin

River is narrow and precipitous at 35～40km section from the junction of Tonegawa river.



Vertical distribution of river width

Basin of Kinugawa river：1760km²

Length of main river：177km

Population in-
Kinugawa river basin : About 550,000

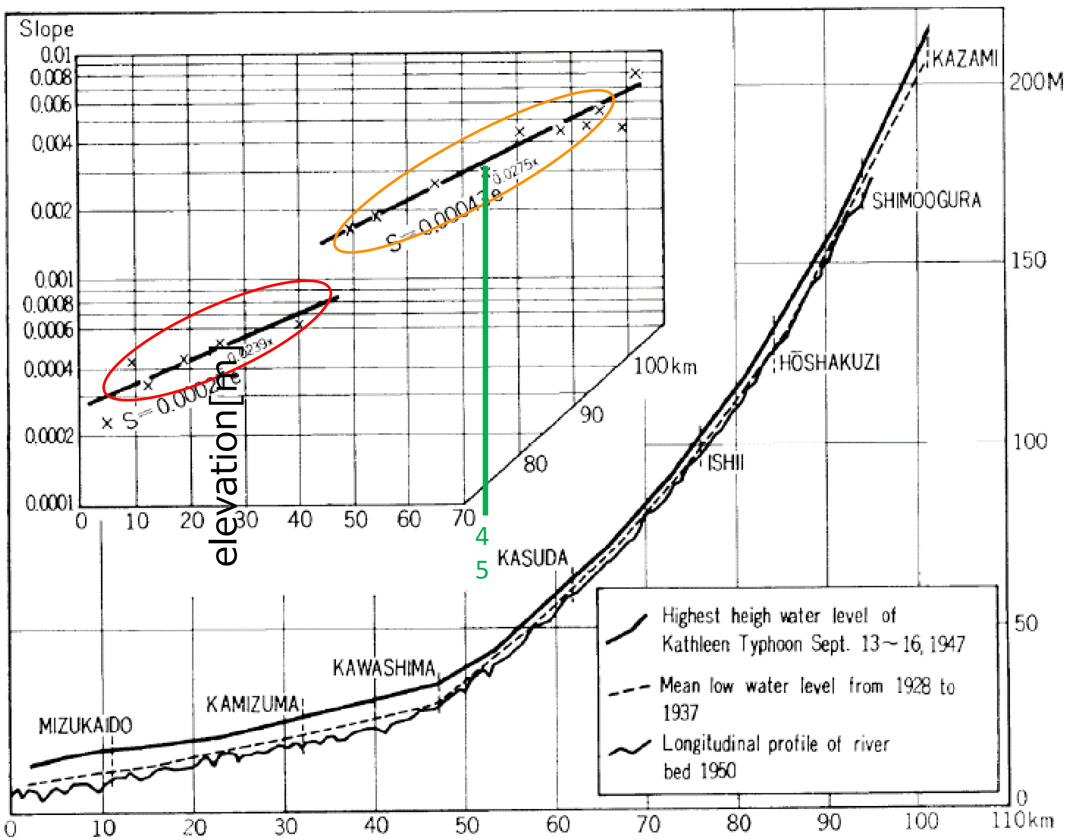
Quoted from『第1回鬼怒川堤防調査委員会資料』Ministry of Land, Infrastructure, Transportation and Tourism

Characteristic of basin : Over 60% is mountains, level ground is about 30%

Heavy rain in the Kanto and Tohoku regions, Sep.2015

Essentials of the Kinugawa River Basin

Eiju Yatsu(1966)：Rock Control in Geomorphology



Longitudinal distance[km](from merging sections of Tone River)

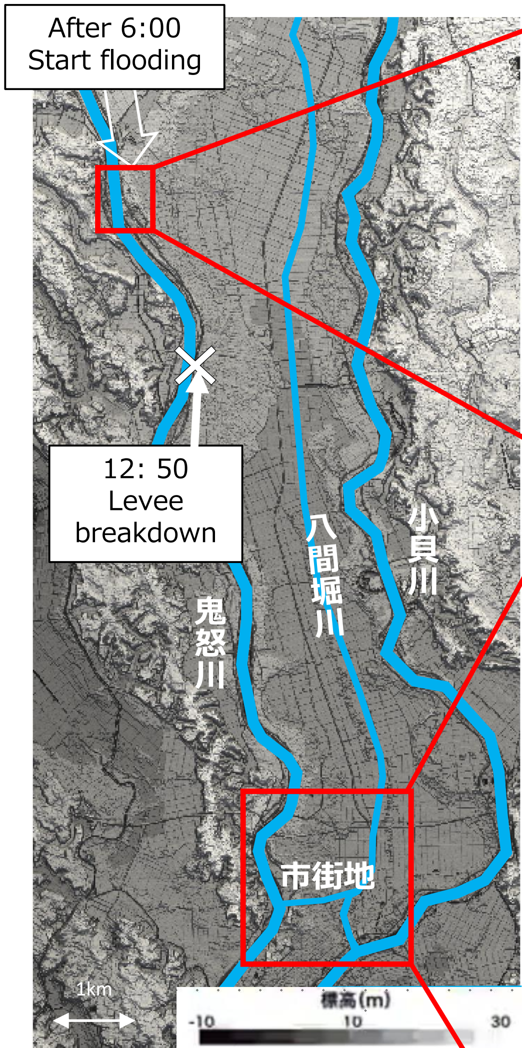
The riverbed profile of the Kinugawa River has two exponential curves.

It pointed out that there are rivers with two exponential curves for the first time in the world (the most of the river longitudinal profile rivers is an exponential curve), the river bed sediment particle size at the place where the river bed longitudinal form folds I revealed that it is changing by Prof. Yatsu.

『Rock Control in Geomorphology(1966)』 by Prof. YaTsu

Heavy rain in the Kanto and Tohoku regions, Sep.2015

Topographic characteristics of inundation area and flood condition of urban area



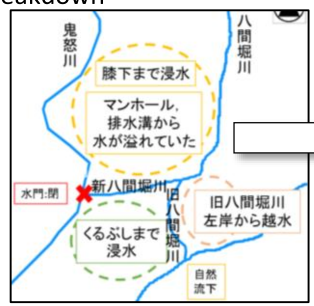
Elevation map around the flood area Created by Geographical Information Authority of Japan (10m DEM)

The natural embankment was excavated by installing a solar panel※ (Captured image on February 2, 2015)
The sandbag was piled up to the original height when the flood happened

※ Ministry of Land, Infrastructure and Transport Kanto Region Development Bureau "About flood damage and restoration situation related to the Kanto-Tohoku heavy rain disaster in September 2015"

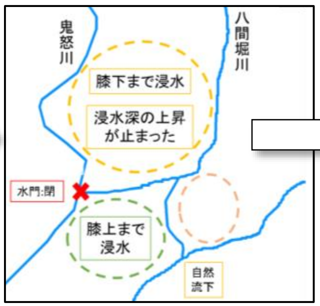
< Inundation situation of urban area by inhabitant hearing survey >

Approximately 2 hours after levee breakdown



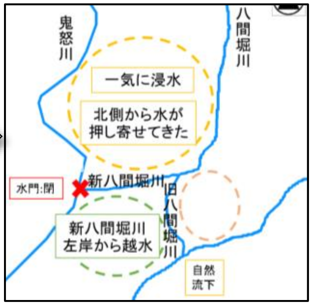
① Inland water flooded in the tributary

Approximately 3 hours after



② the depth of immersion stopped

Approximately 8 hours after



③ Flood water from the main stream reached

→ The habitants mistake the Immersions from the main steam for flood water and then too late to escape .

Heavy rain in the Kanto and Tohoku regions, Sep.2015

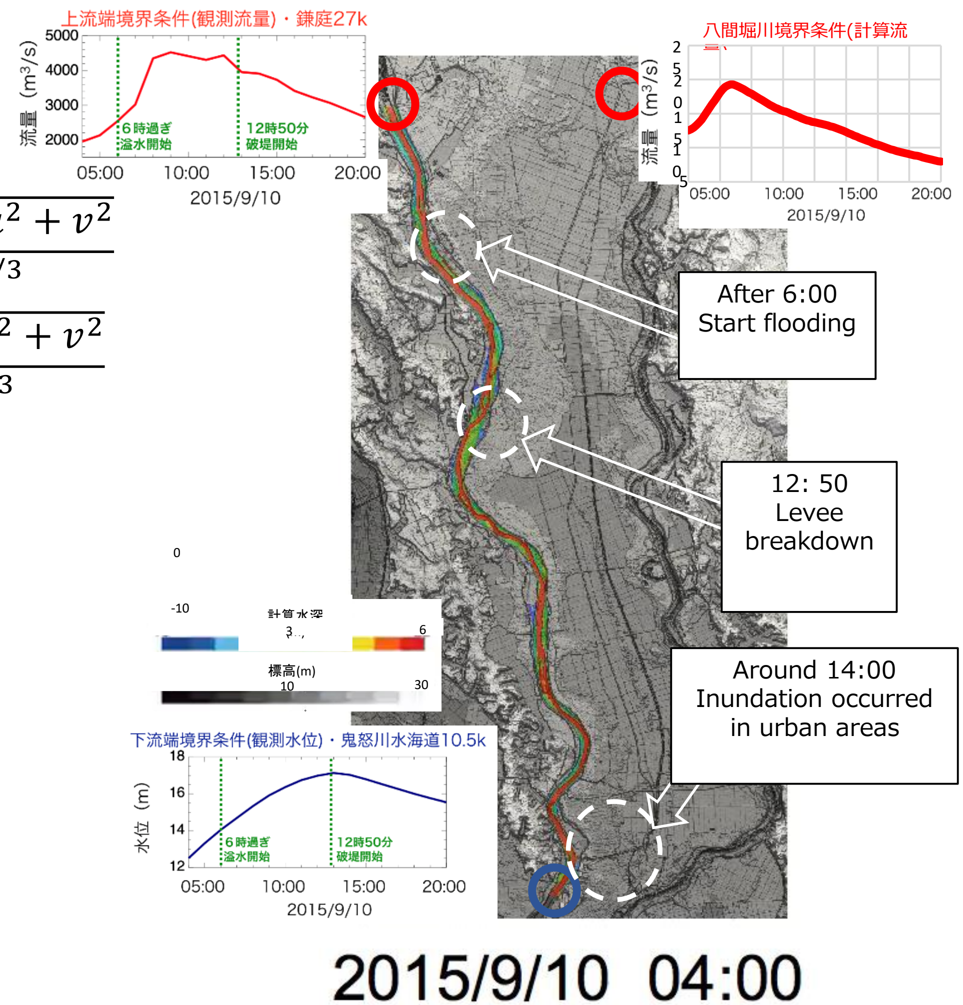
Reproduction of inundation situation in Joso city by flood inundation analysis

Basic equations (shallow water equations

$$\frac{\partial h}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0$$

$$\frac{\partial M}{\partial t}+\frac{\partial}{\partial x}\left(\frac{M^2}{h}\right)+\frac{\partial}{\partial y}\left(\frac{MN}{h}\right)=-gh\frac{\partial H}{\partial x}-\frac{gn^2u\sqrt{u^2+v^2}}{h^{1/3}}$$

$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}\left(\frac{MN}{h}\right)+\frac{\partial}{\partial y}\left(\frac{N^2}{h}\right)=-gh\frac{\partial H}{\partial y}-\frac{gn^2v\sqrt{u^2+v^2}}{h^{1/3}}$$



M, N : *x, y* Flow flux *t* : Time coordinates *x, y* : Plane coordinates *h* : Depth 、 *g* : Gravity *n* : Roughness Coefficient *H* : Water level *u, v* : *x, y* flow velocity

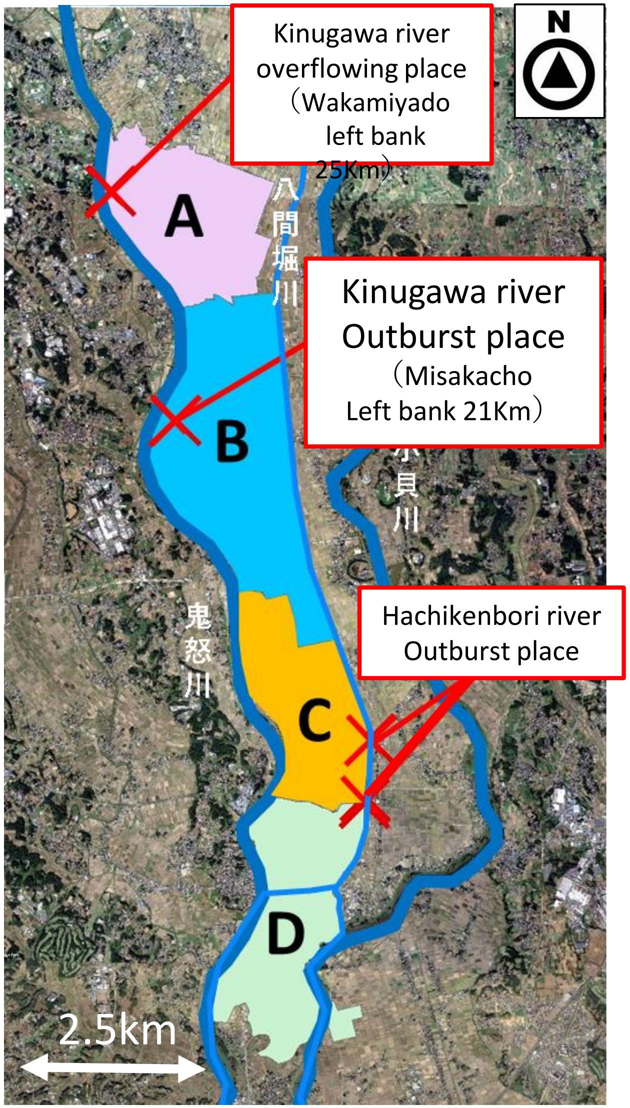
Differentiated equations by Leap-frog method

$\Delta x=\Delta y=10$m、 $\Delta t=0.2$s

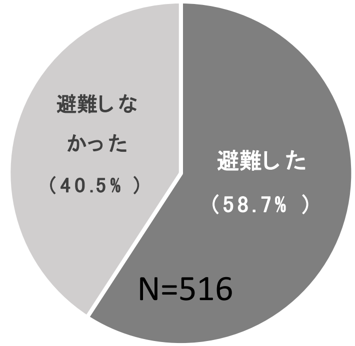
The roughness coefficient of a river channel and a flood plain was equally set to 0.03 [m-1/3 s]

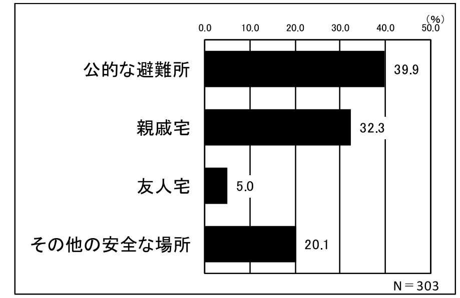
Research on the behavior of evacuation

Evacuation situation by district at the time of disaster

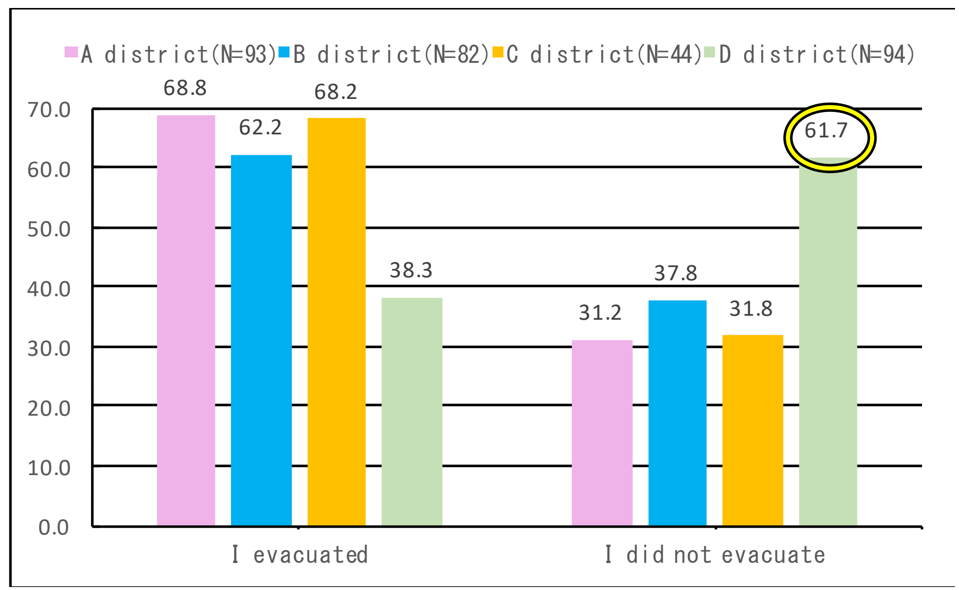


District division map of survey





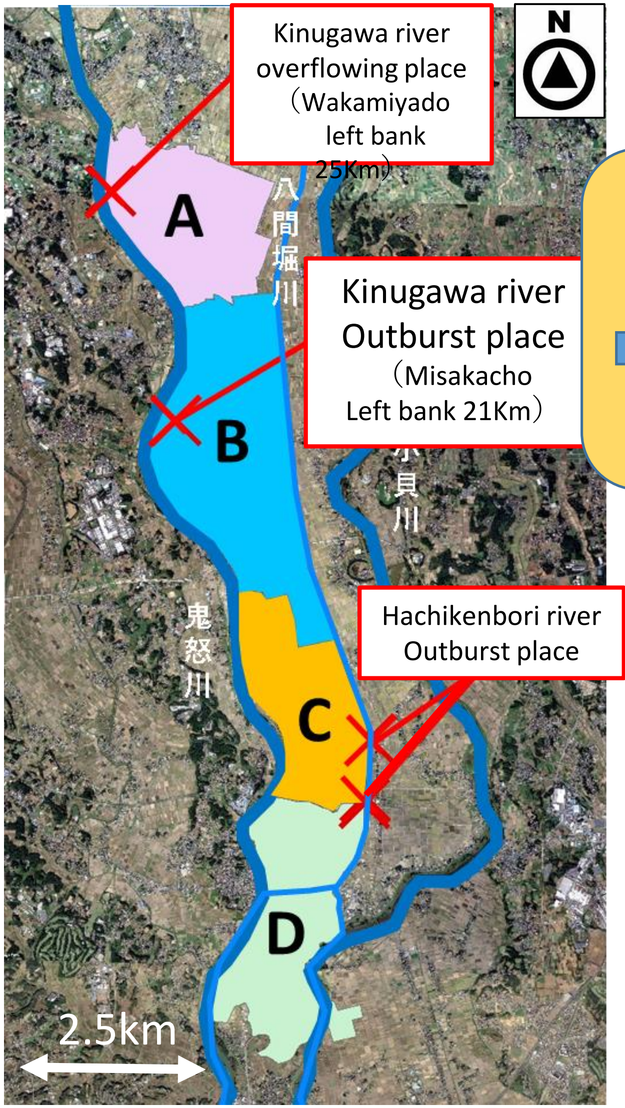
→There were 59% of the entire survey households evacuated to shelters, and another 41% were at home without evacuation.



Most residents in district D did not evacuate.

Research on the behavior of evacuation

District A: Around the overflow area of the embankment of the Kinugawa River

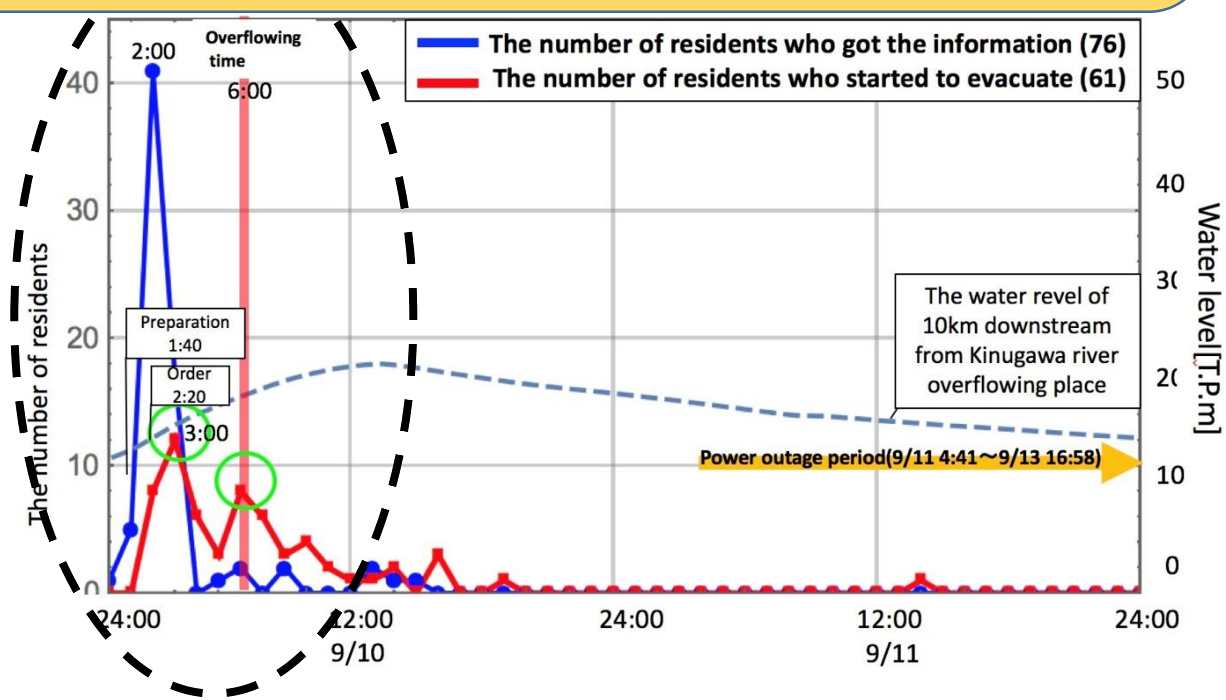


District division map of survey

Many residents started to evacuate at

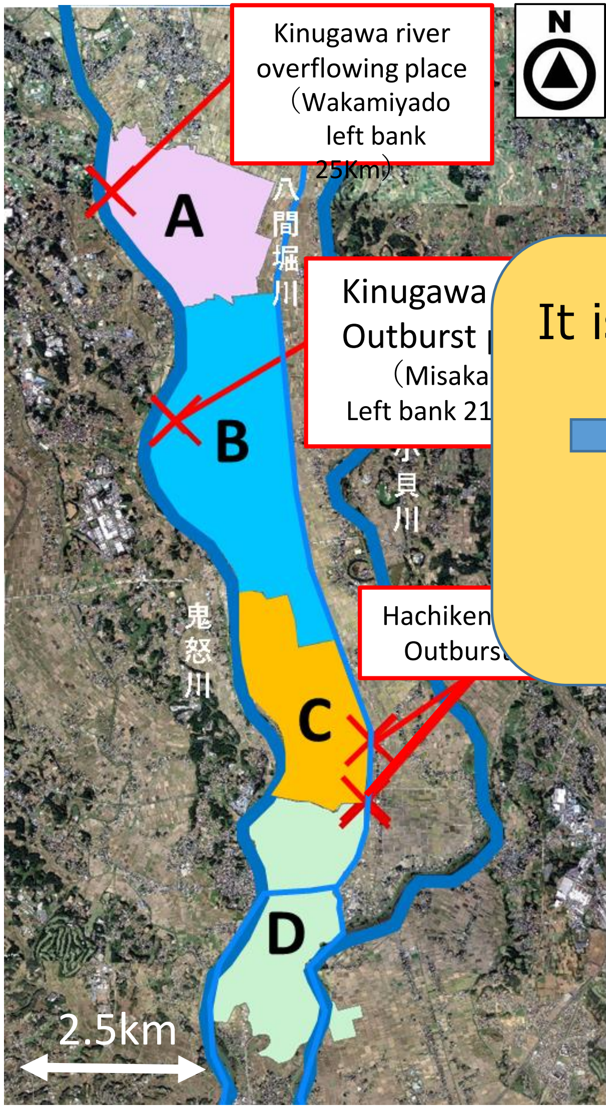
Residents in District A can easily recognize the risk of flooding,

Residents evacuated immediately after getting evacuation information



Research on the behavior of evacuation

District B: Around the broken part of the embankment of the Kinugawa River

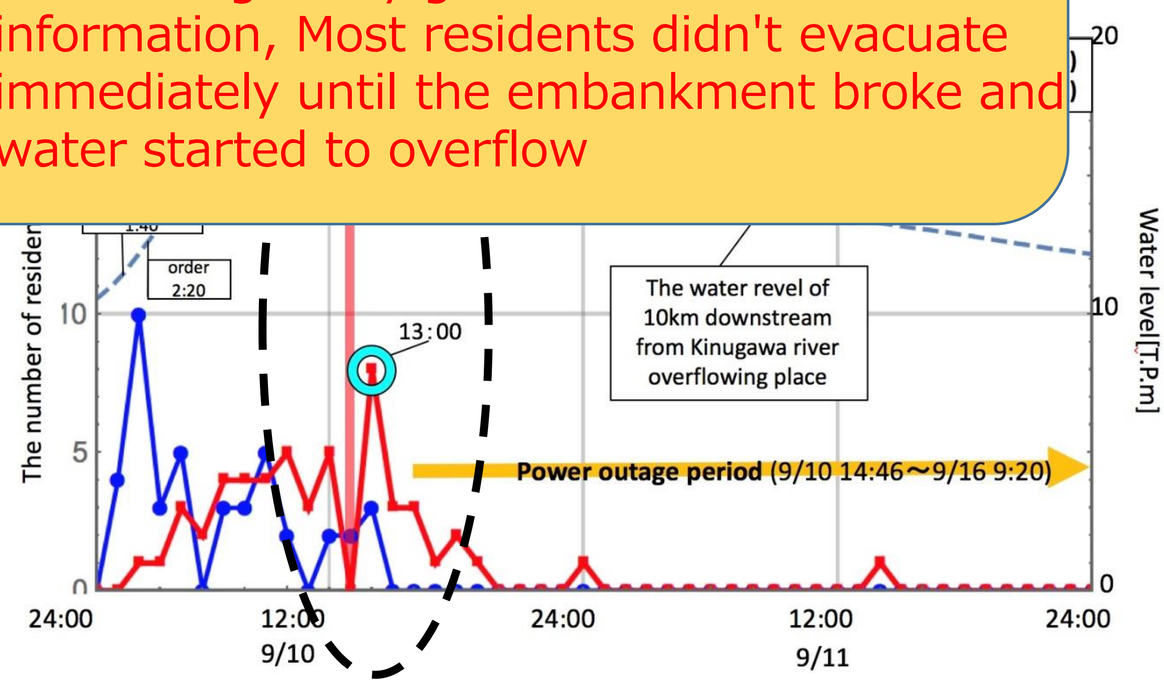


District division map of survey

・Many residents started to evacuate right after the embankment of the Kinugawa river collapsed (at 1pm 10 Sep)

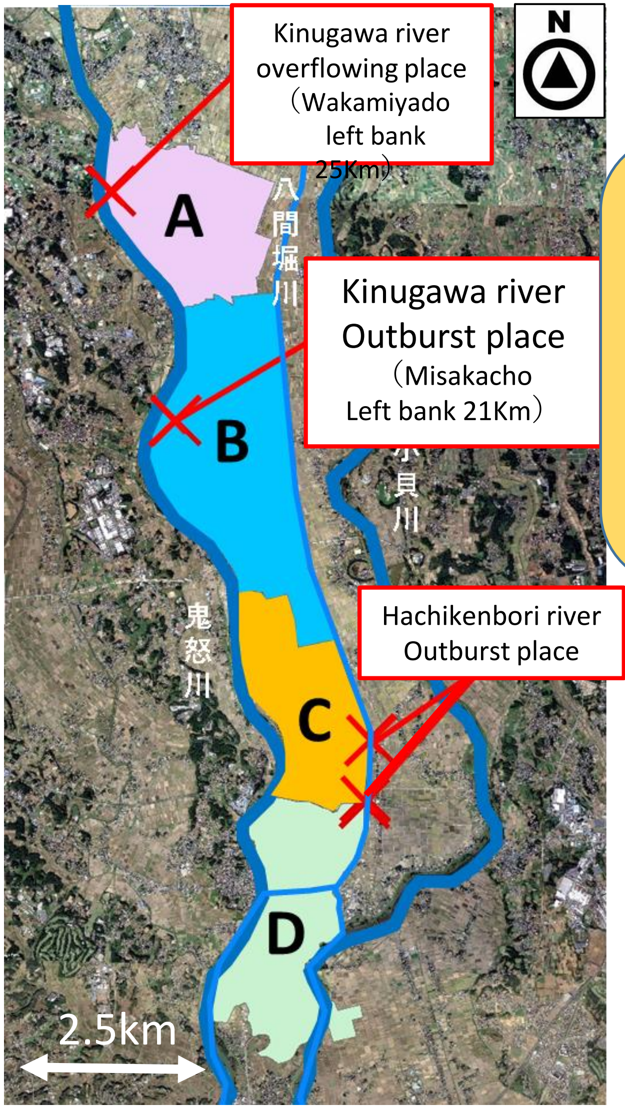
It is hard to imagine where the embankment break down

Even though they got a evacuation information, Most residents didn't evacuate immediately until the embankment broke and water started to overflow



Research on the behavior of evacuation

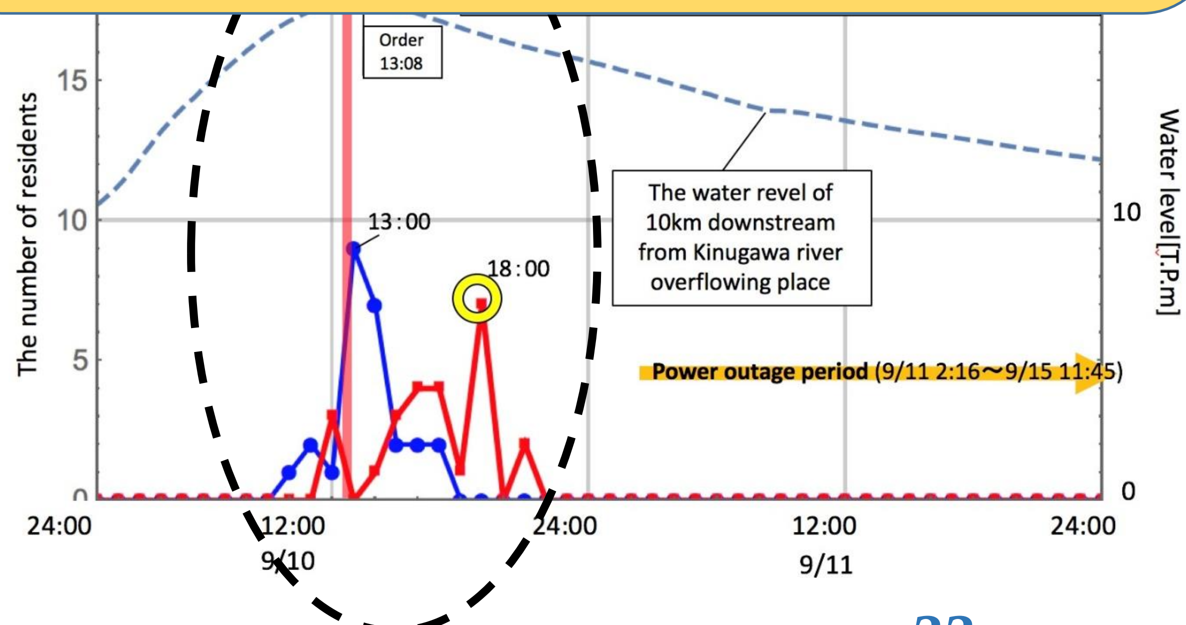
District C: Between the broken part of the embankment of the Kinugawa River and a city area



District division map of survey

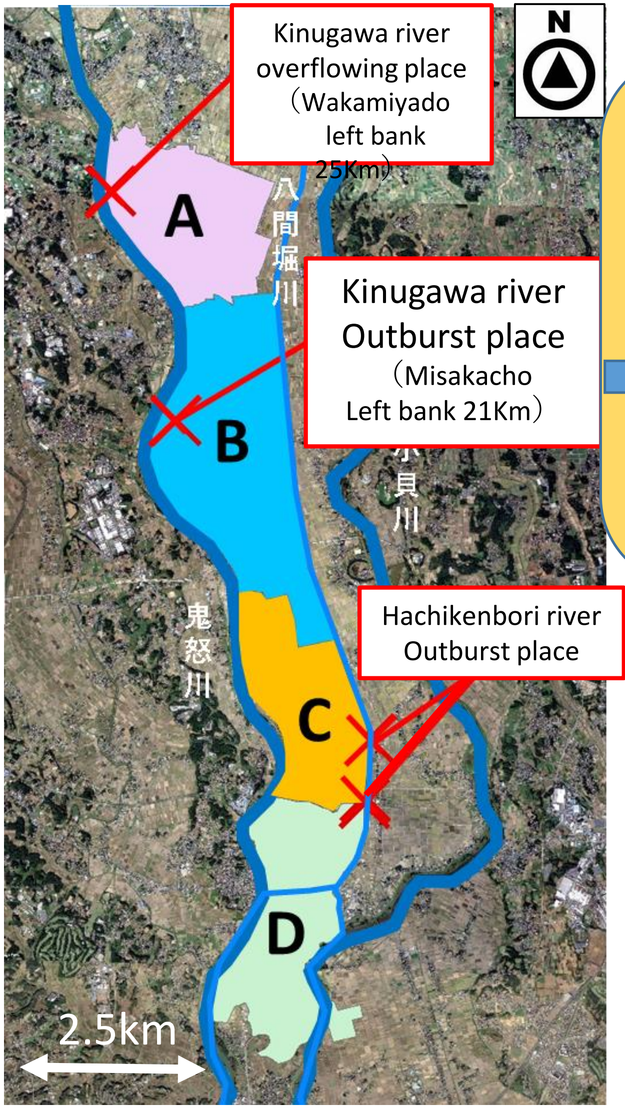
It is hard to imagine where the embankment breaks down and It is away from area where embankment broke

Even getting evacuation information, Most residents did not evacuate before when overflow arrive



Research on the behavior of evacuation

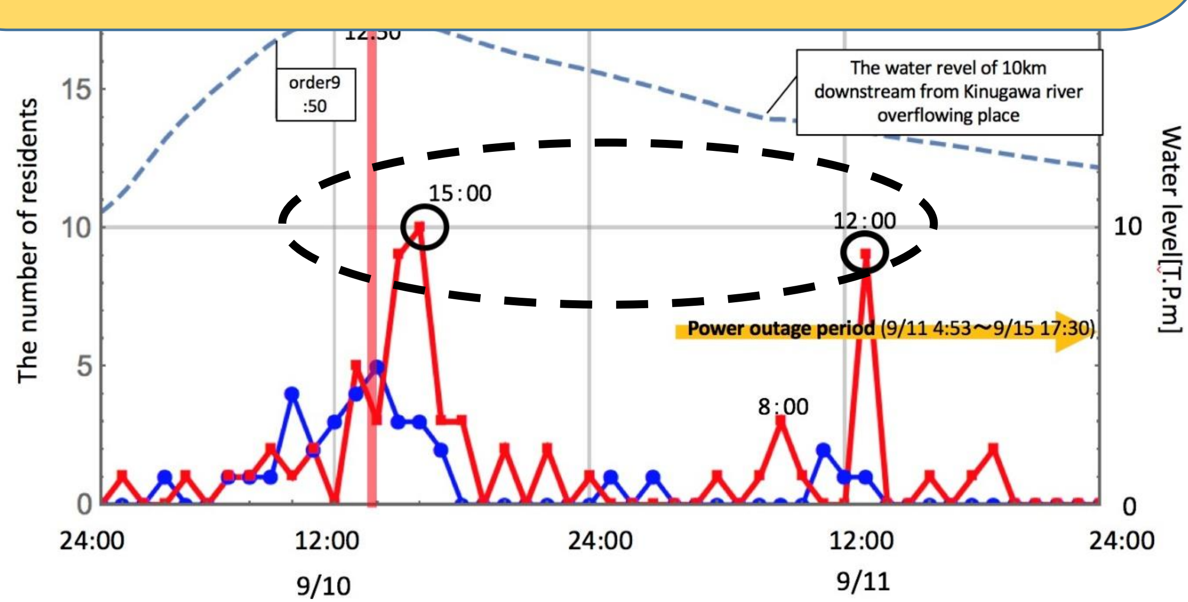
District D: A city area of Mitsukaido



District division map of survey

It is away from area where embankment broke and Inundation inside the levee of the Hachikenbori river

There were few residents compared with other districts. There were residents who evacuated when inundation occurred and the others evacuated the next day because of power outage.



Research on the behavior of evacuation

Acquisition of the disaster information and evacuation situation (All the areas that surveyed)

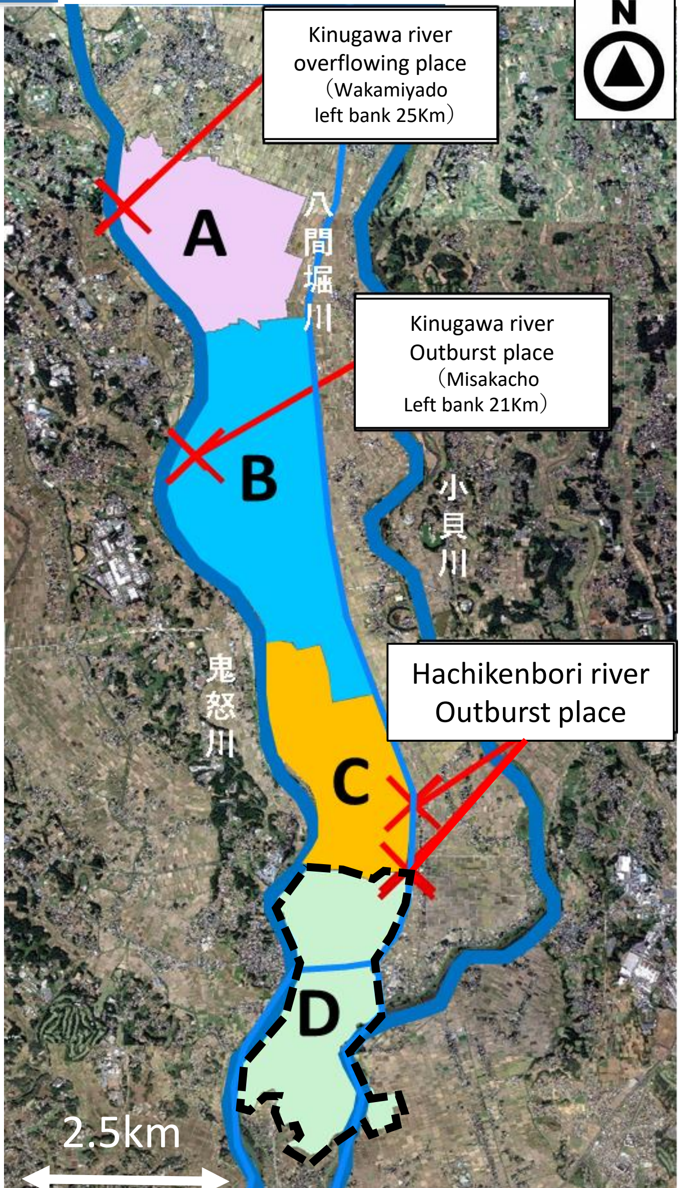
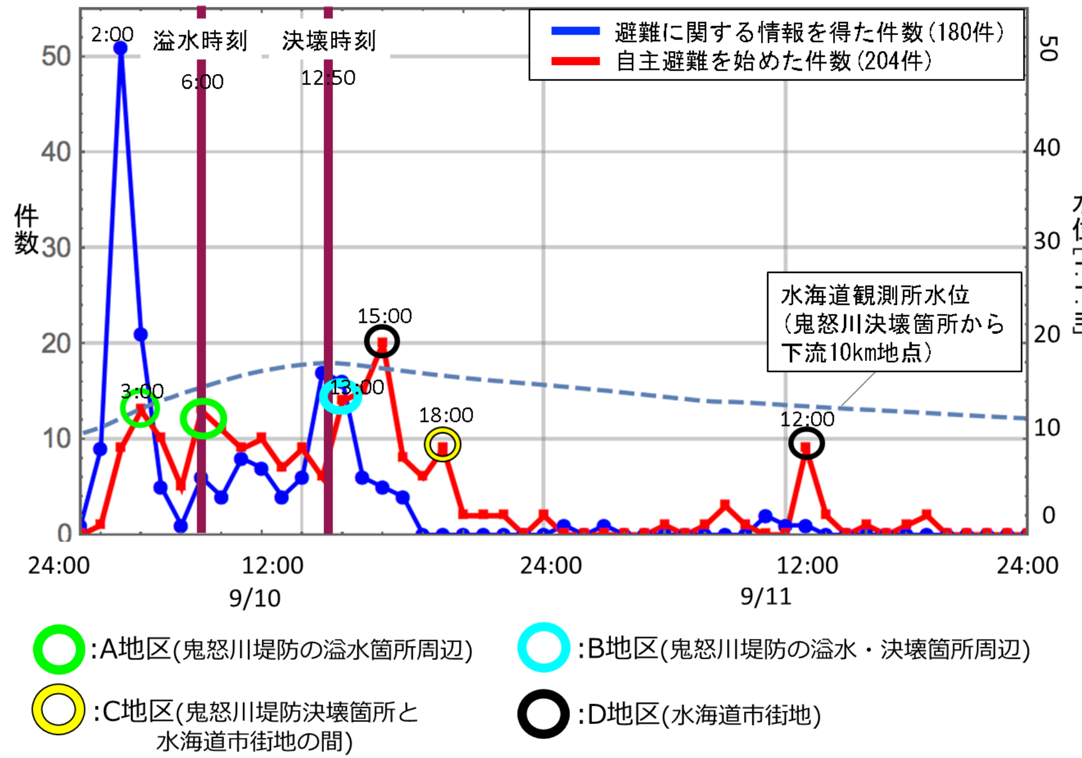


Figure of division of the hearing point



A District : Residents recognized a risk of the inundation
B, C District : It is uncertain and which area seems to be flooded and it is hard to recognize where the rip of dike occurs
D District : The possibility that the inland waters flooding caused by the flooding of affluent had an influence on to a refuge action from the rip spot of the Kinugawa dike if a long time ago

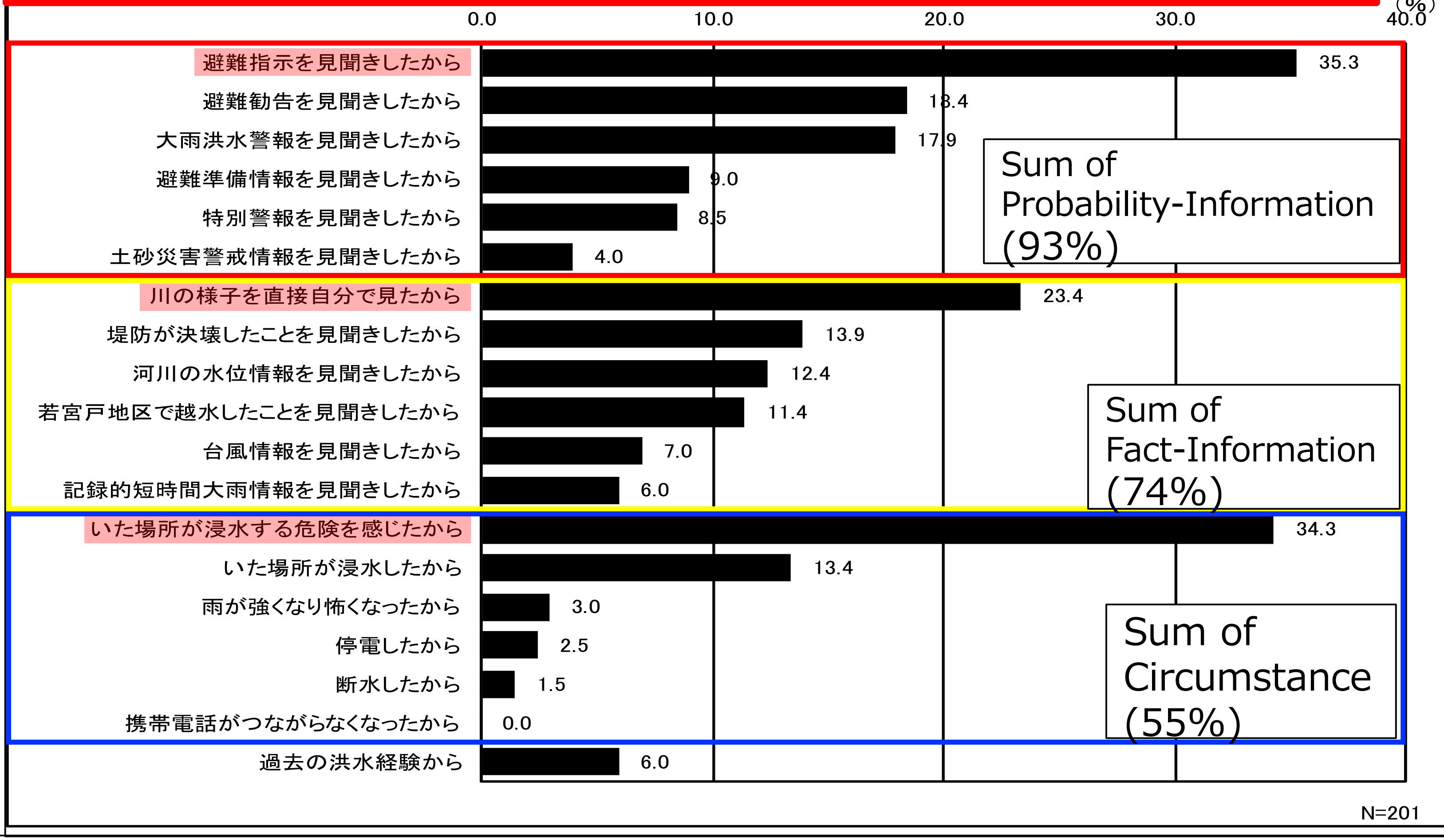
I discovered that there was a difference about time when residents evacuate after getting evacuation inform.

Evacuation Triggers (Multiple Answers)

Evacuation Triggers (Multiple Answers)

Fact-Information occurred and observed （ex. Information of rainfall and water level）

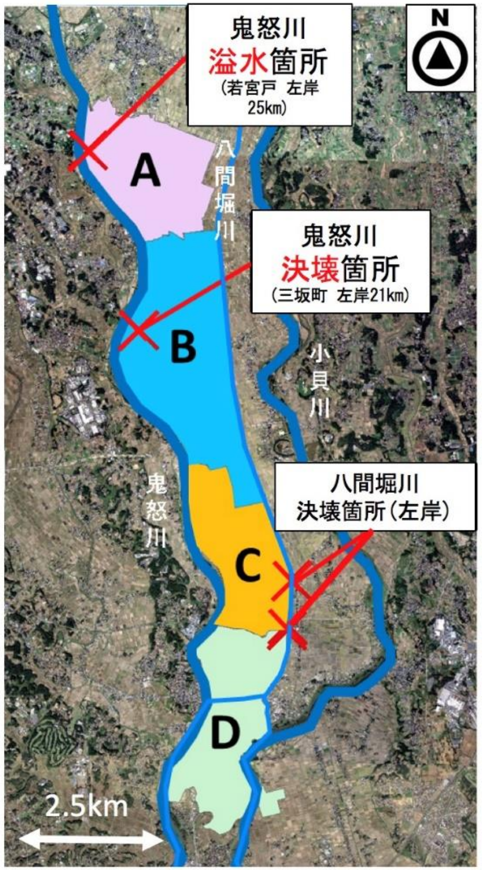
Probability-Information：The outbreak probability of some kind of phenomena being high(ex. Emergency warning、Evacuation order)



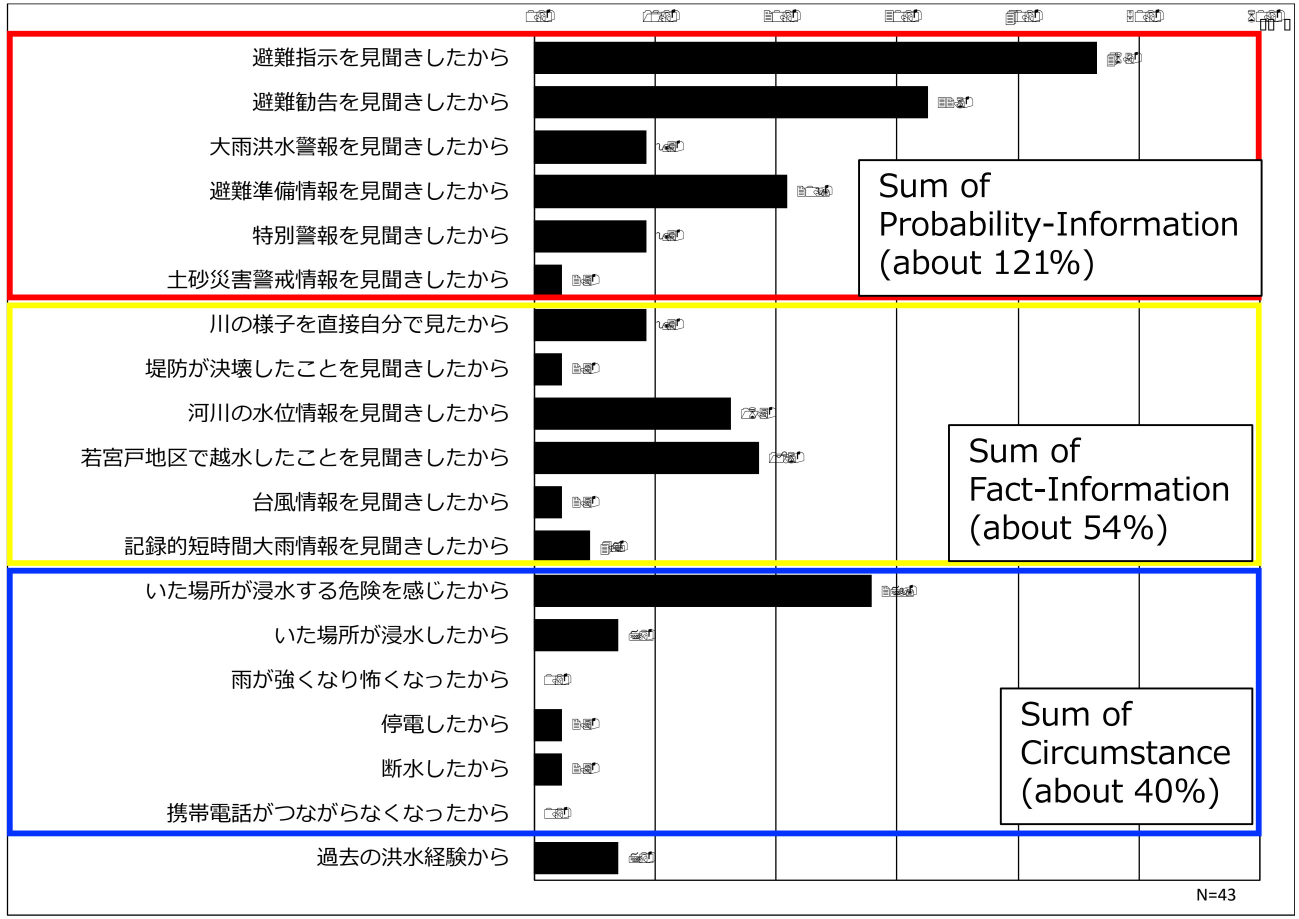
Evacuation triggers had most totals of the probability information and was 93% followed by fact information, a surrounding.

Evacuation Triggers (Multiple Answers)

A District（Around Overtopping point）Evacuation Triggers



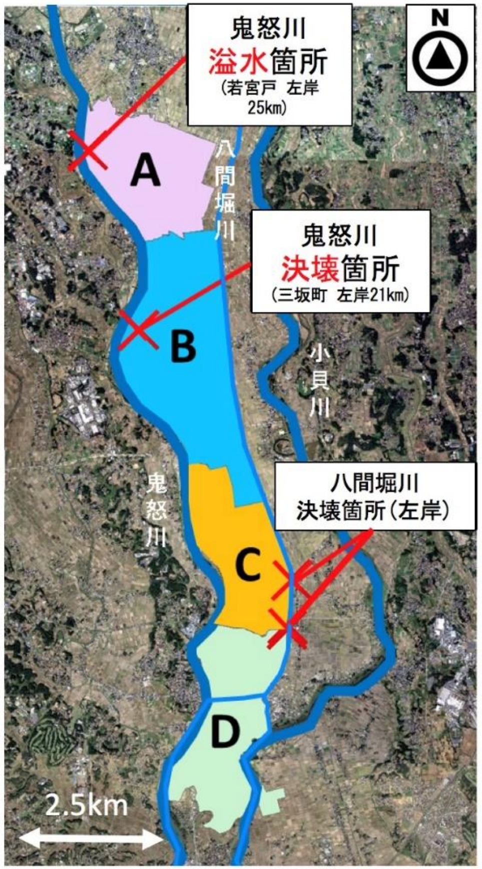
ヒアリング実施箇所の区分図



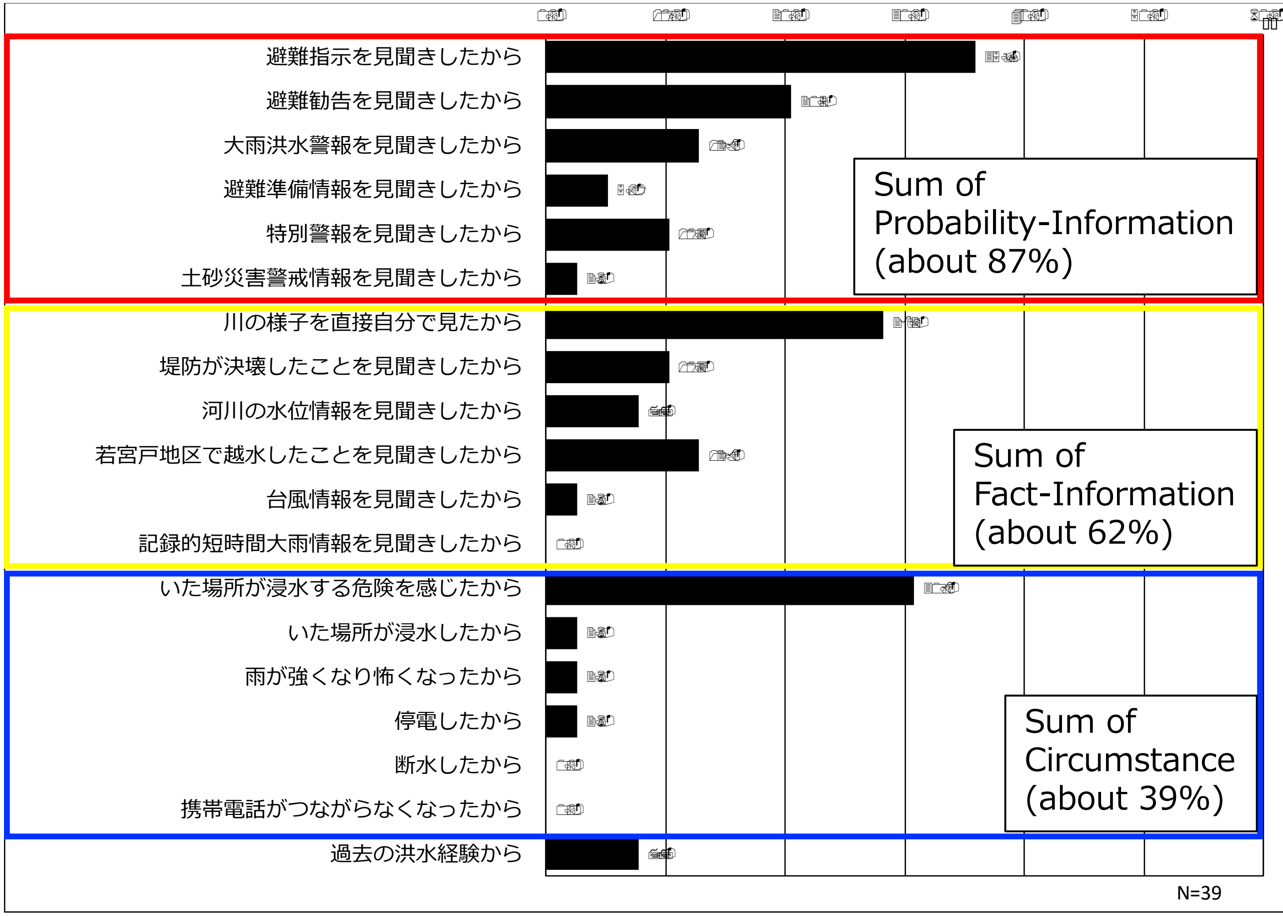
- Evacuation Triggers of the A district has the most **probability information**.
- Residents evacuated through probability information in the A district that could usually recognize a risk of the inundation easily.

Evacuation Triggers (Multiple Answers)

B District（Around Overtopping point）Evacuation Triggers



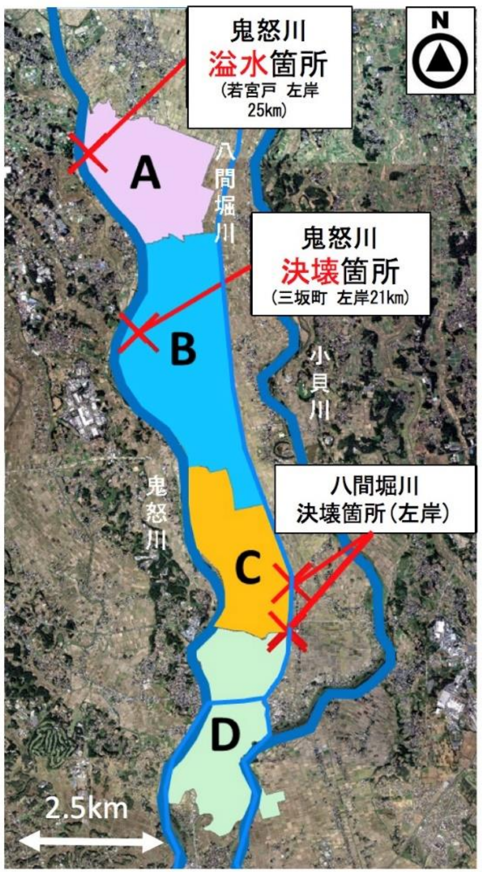
ヒアリング実施箇所の区分図



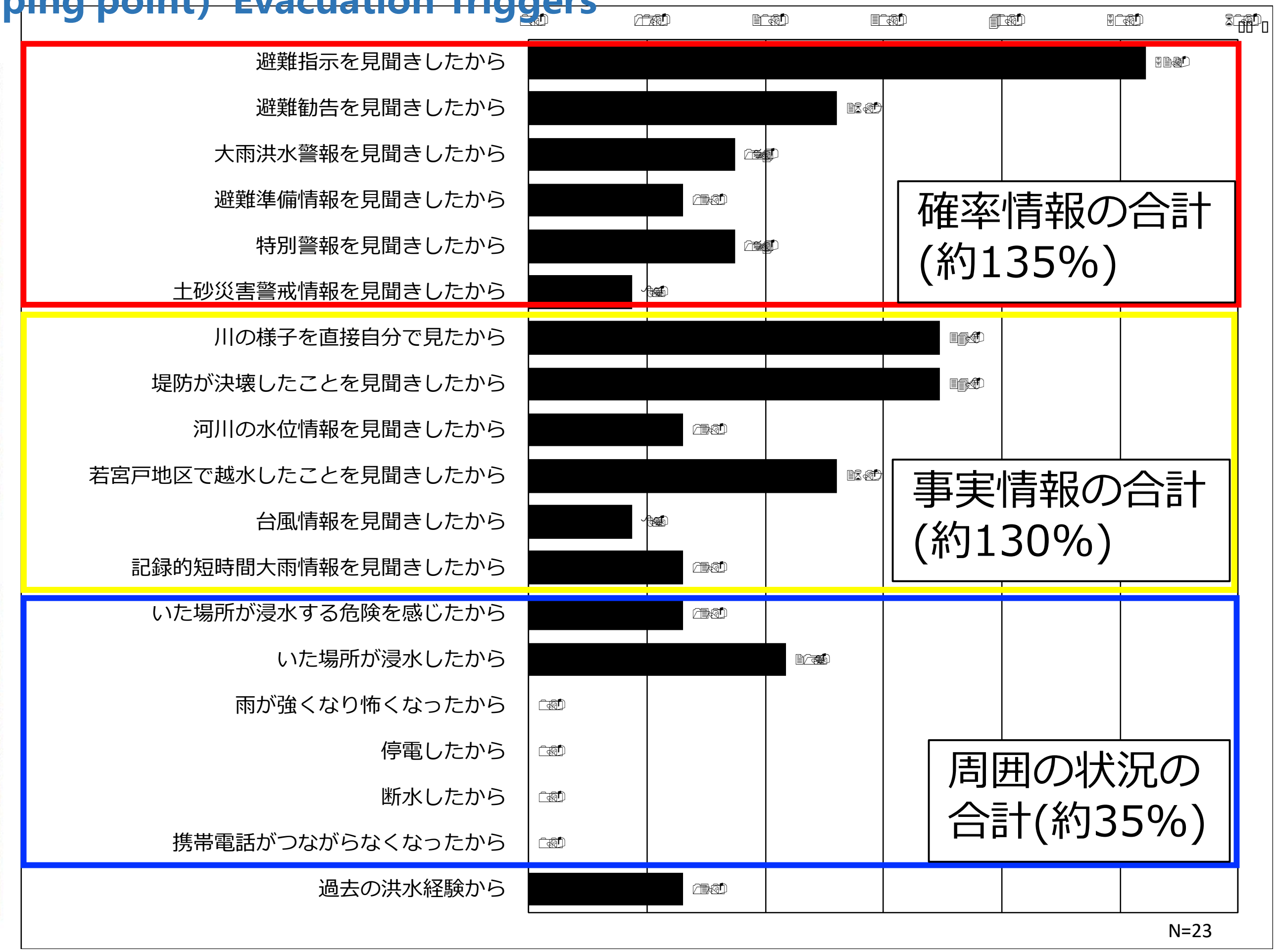
- Evacuation Triggers of the B district has most **probability information**
- Evacuation order(13:08) is just after the rip of the dike(12:50), and the peak of the evacuation is after a rip. Evacuation order that received a rip might lead to the evacuation.

Evacuation Triggers (Multiple Answers)

C District (Around Overtopping point) Evacuation Triggers



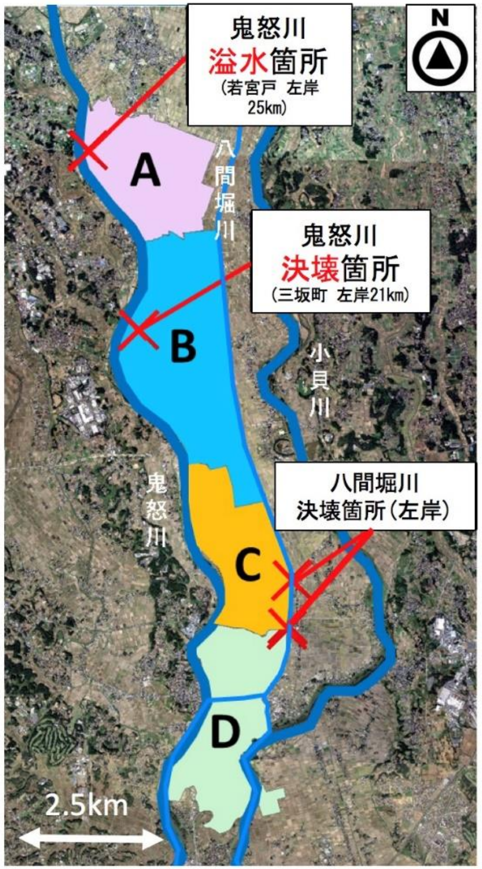
ヒアリング実施箇所の区分図



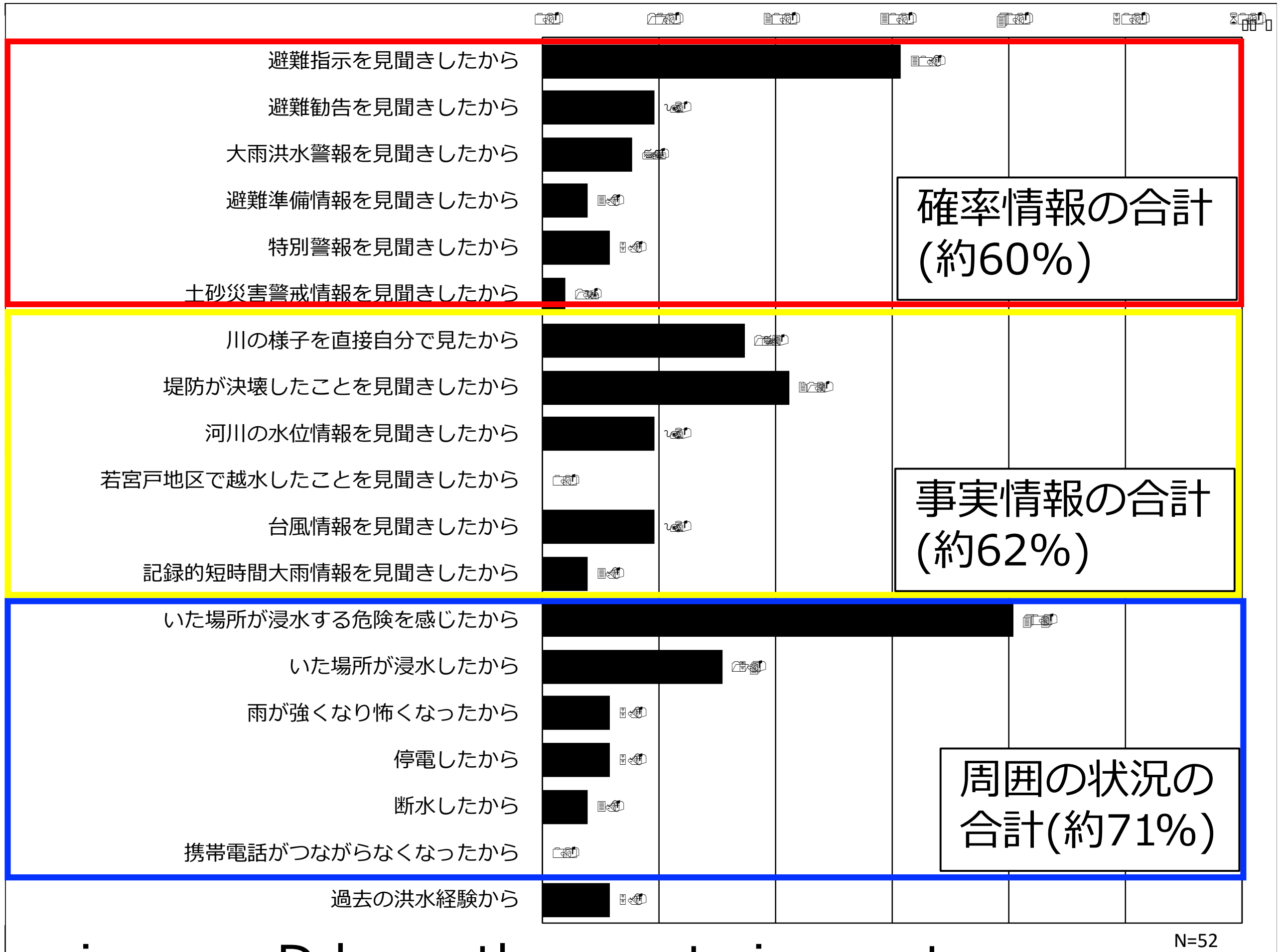
- factual information and probability information(5:5)
- It took time for residents to evacuate after they got the information. Focusing on total amount for factual information and probability information, residents based to evacuate on these kinds of information.

Evacuation Triggers (Multiple Answers)

D District (Around Overtopping point) Evacuation Triggers



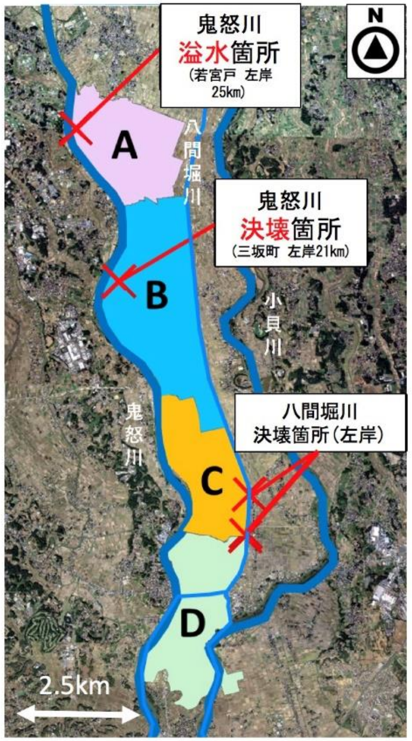
ヒアリング実施箇所の区分図



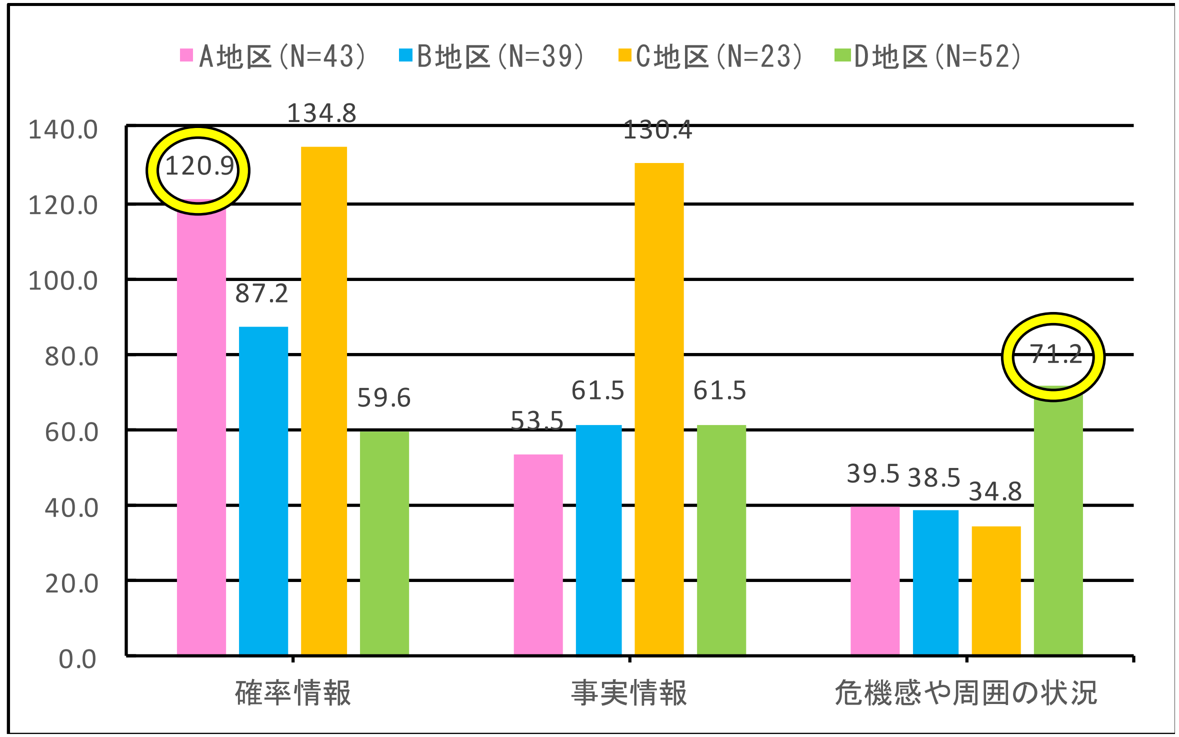
・Trigger of evacuation in area D have the most circumstances

・Trigger of evacuation in district D is changing circumstance rather than probability-information and factual-information.

Evacuation Triggers (Multiple Answers)



ヒアリング実施箇所の区分図



District A : District A has the most probability information. It seems that probability information is helpful for evacuation , because district could recognize easily flood risk.

District B : District B has the most probability information.The trigger is evacuation order that ordered right after a river bank breach.

District C : Probability information and factual information are almost the same rate. Residents who live in district C evacuated from judging with plurality of information.

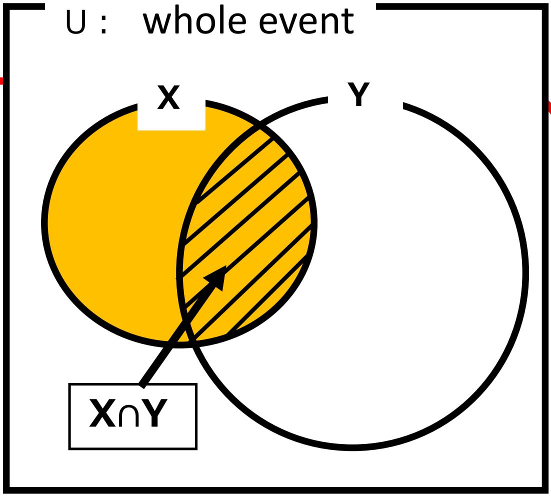
District D : Trigger of evacuation in district D is changing circumstance rather than probability-information and factual-information.

information that was effective for evacuation

<Conditional Probability>

Possibility of event Y is happened by Event X condition.Then, it call that possibility of Event Y's condition what is based on Event X

$$P(Y|X) = \frac{P(X \cap Y)}{P(X)} \quad ①$$



<Multiplicative theorem>

$$P(Y|X) = \frac{P(X \cap Y)}{P(X)} \quad ① \qquad P(X|Y) = \frac{P(X \cap Y)}{P(Y)} \quad ②$$

From①and②

$$P(X \cap Y) = P(Y|X)P(X) = P(X|Y)P(Y) \quad ③$$

Information of evacuation effect

$$p(\mathrm{x} \cap \mathrm{y}) = p(\mathrm{x}|\mathrm{y})\mathrm{p}(\mathrm{y}) = (\mathrm{y}|\mathrm{x})p(\mathrm{x}) \quad ③$$

In ③ equation, plus x of all possible X,
from the definition of probability, $\sum_x p(x|y) = 1$
Therefore ③ equation become

$$p(\mathrm{y}) = \sum_x p(y|x)p(x) \quad ④$$

Bayes' theorem

And ③ equation divided by ④ equation,

Prior probability

Posterior probability

$$p(x|y) = \frac{p(y|x)p(x)}{\sum_{\mathrm{x}} p(y|x)p(x)}$$

Information of evacuation effect

$$p(\mathrm{x} \cap \mathrm{y}) = p(\mathrm{x}|\mathrm{y})\mathrm{p}(\mathrm{y}) = (\mathrm{y}|\mathrm{x})p(\mathrm{x}) \quad ③$$

In ③ equation, plus x of all possible X,

from the definition of probability, $\sum_x p(x|y) = 1$

Therefore ③ equation become

$$p(\mathrm{y}) = \sum_x p(y|x)p(x) \quad ④$$

x：The residents evacuated、 y：The residents heard the information

$p(y|x)$： The ratio of the information that the residents evacuated heard (possibility)

Prior probability

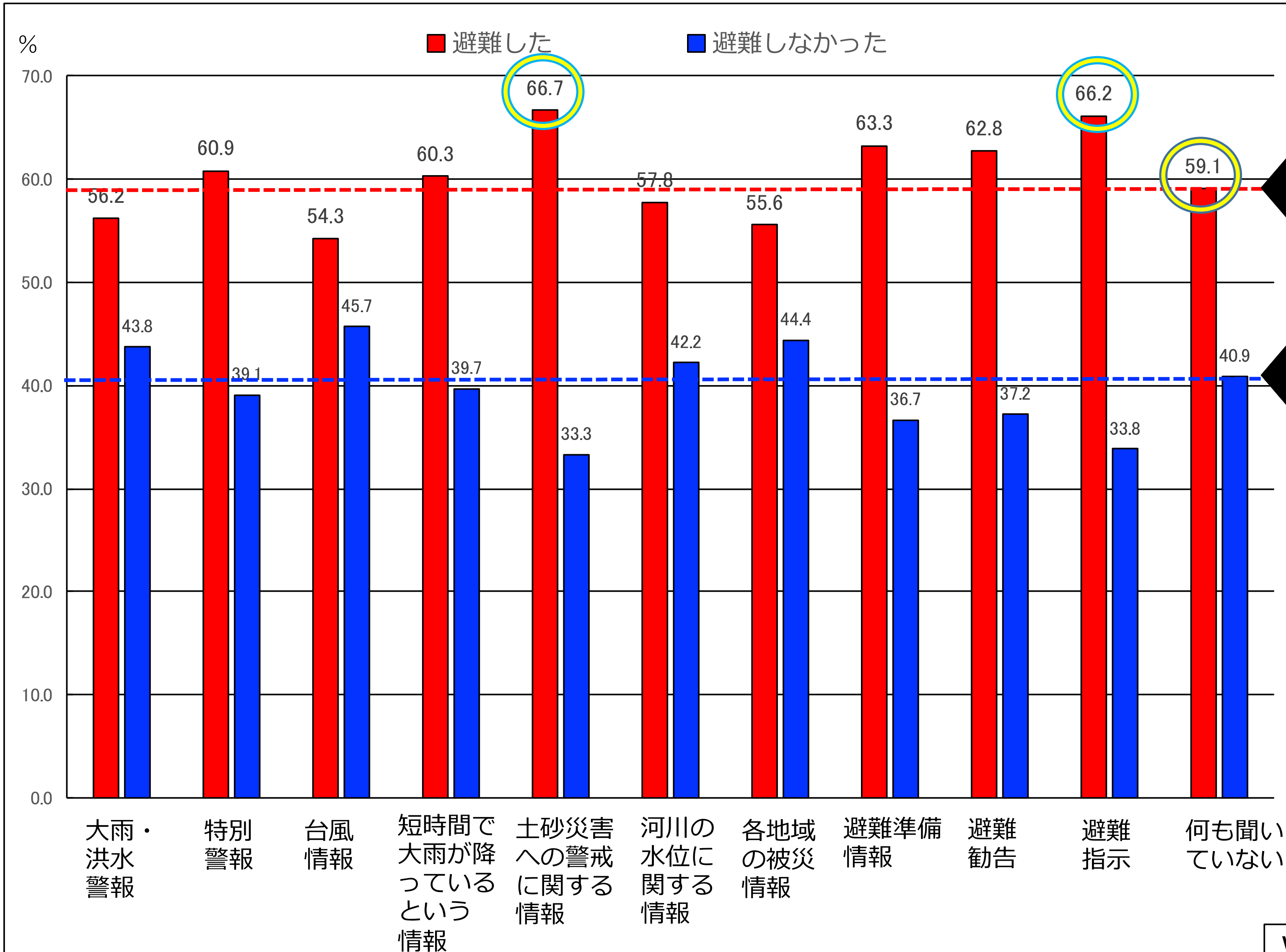
Posterior probability

$$p(x|y) = \frac{p(y|x)p(x)}{\sum_{\mathrm{x}} p(y|x)p(x)}$$

$p(x|y)$：The ratio of the residents that heard the information

Posterior probability of the case that Prior probability of the residents evacuated is 59.2％ (actual ratio of the residents evacuated in all survey)

WE DON'T KNOW!!



The ratio The residents evacuated 59.2%

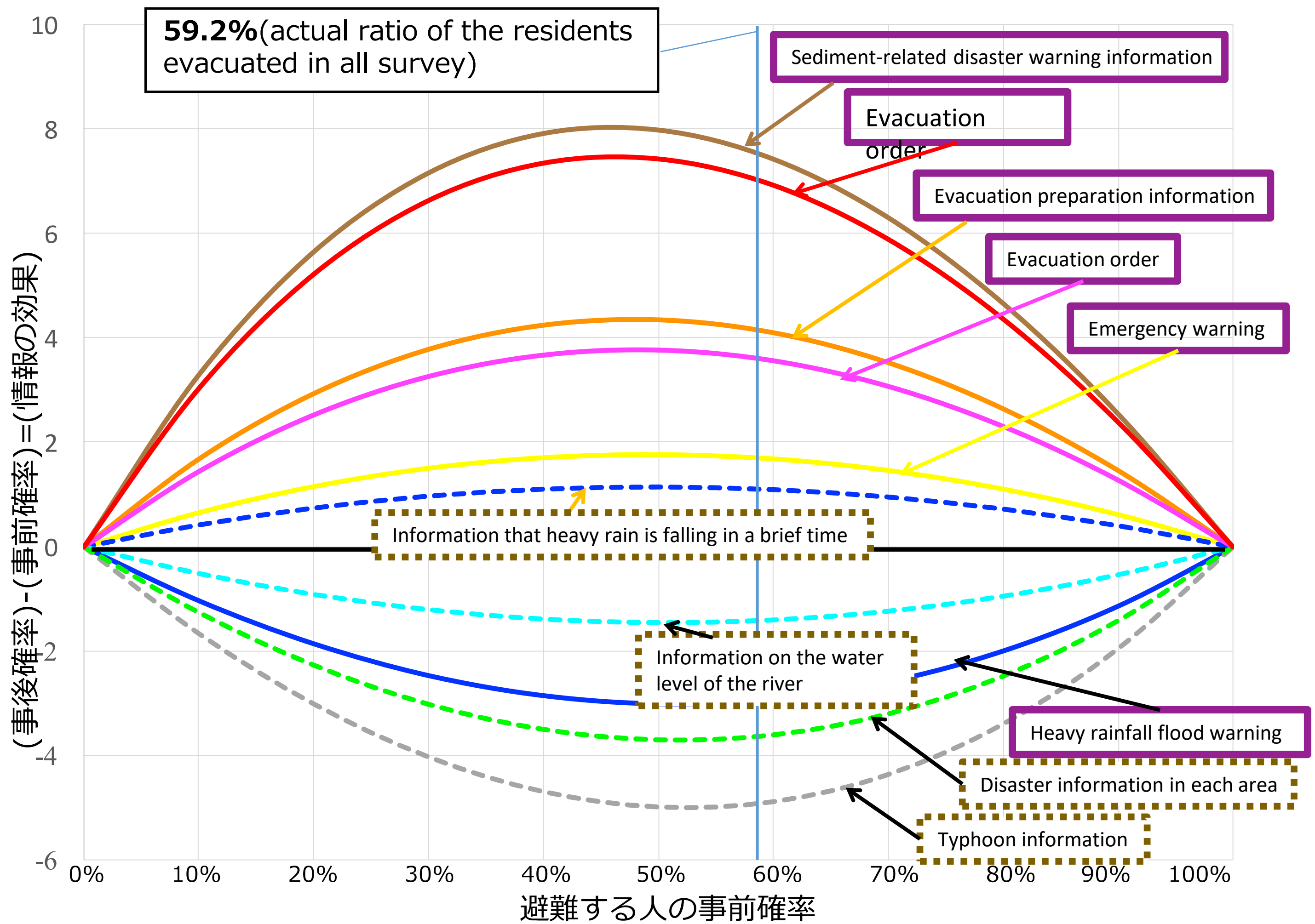
The ratio The residents Not evacuated 40.8%

We got Information

A whole Joso-shi The relationship between Posterior probability of the residents evacuated that got information and prior probability of the residents evacuated

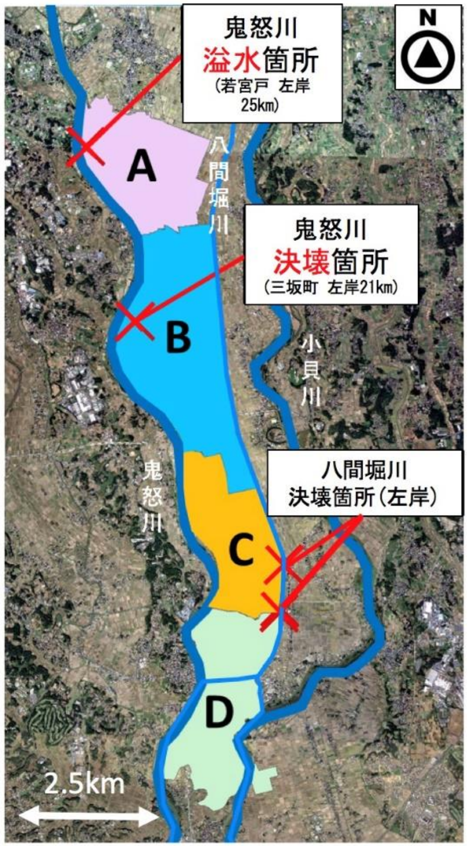
Factual Information : The information that occured and observed information by the time

Probability Information : Enhancing event of possibility after that event

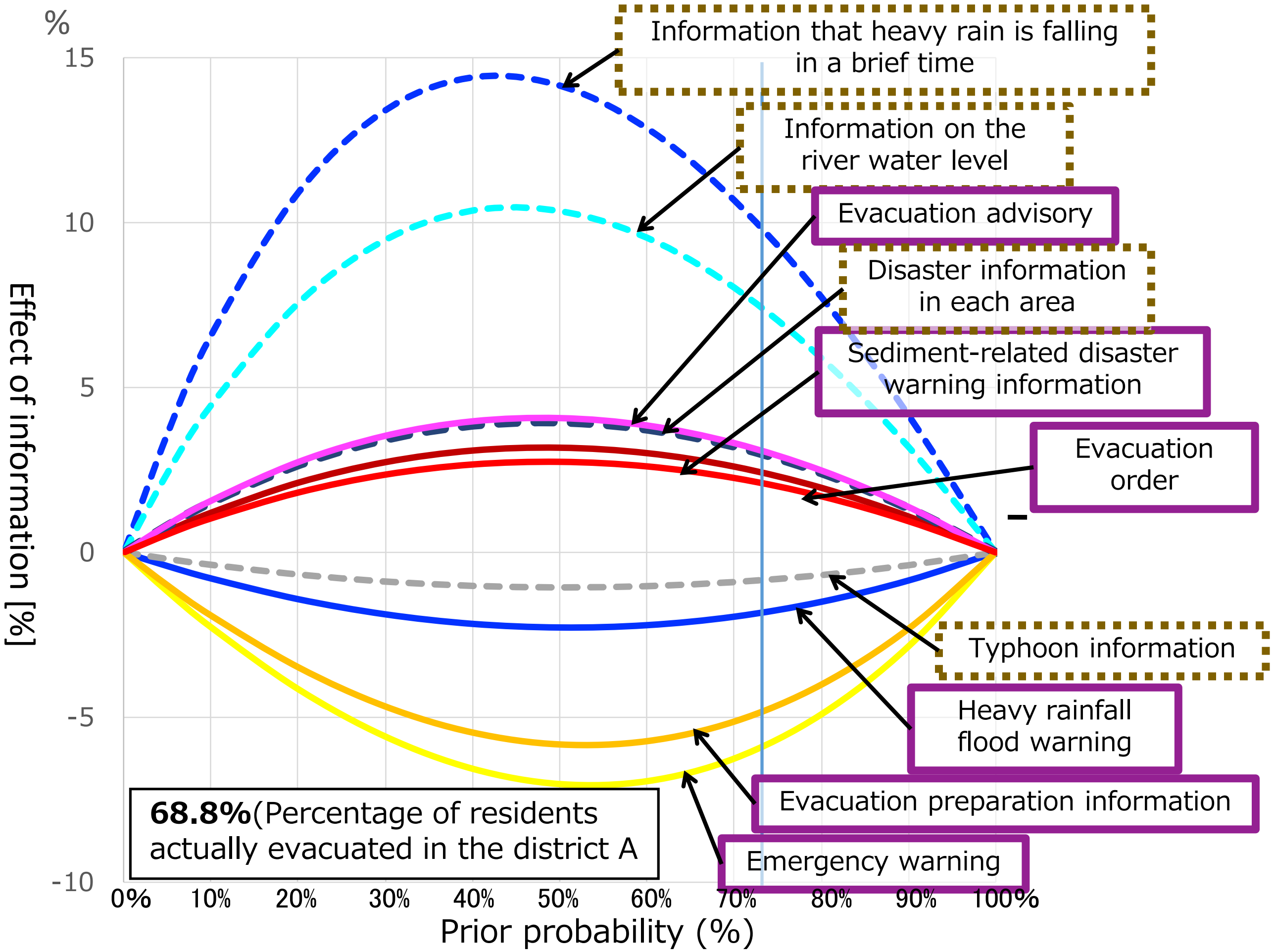


→In a whole Joso-shi, the probability information is more effective than the factual information

District A(Around the overflow area of the embankment)

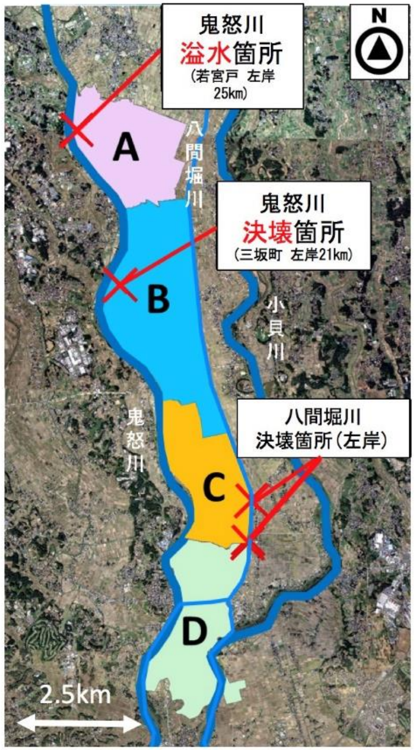


ヒアリング実施箇所の区分図

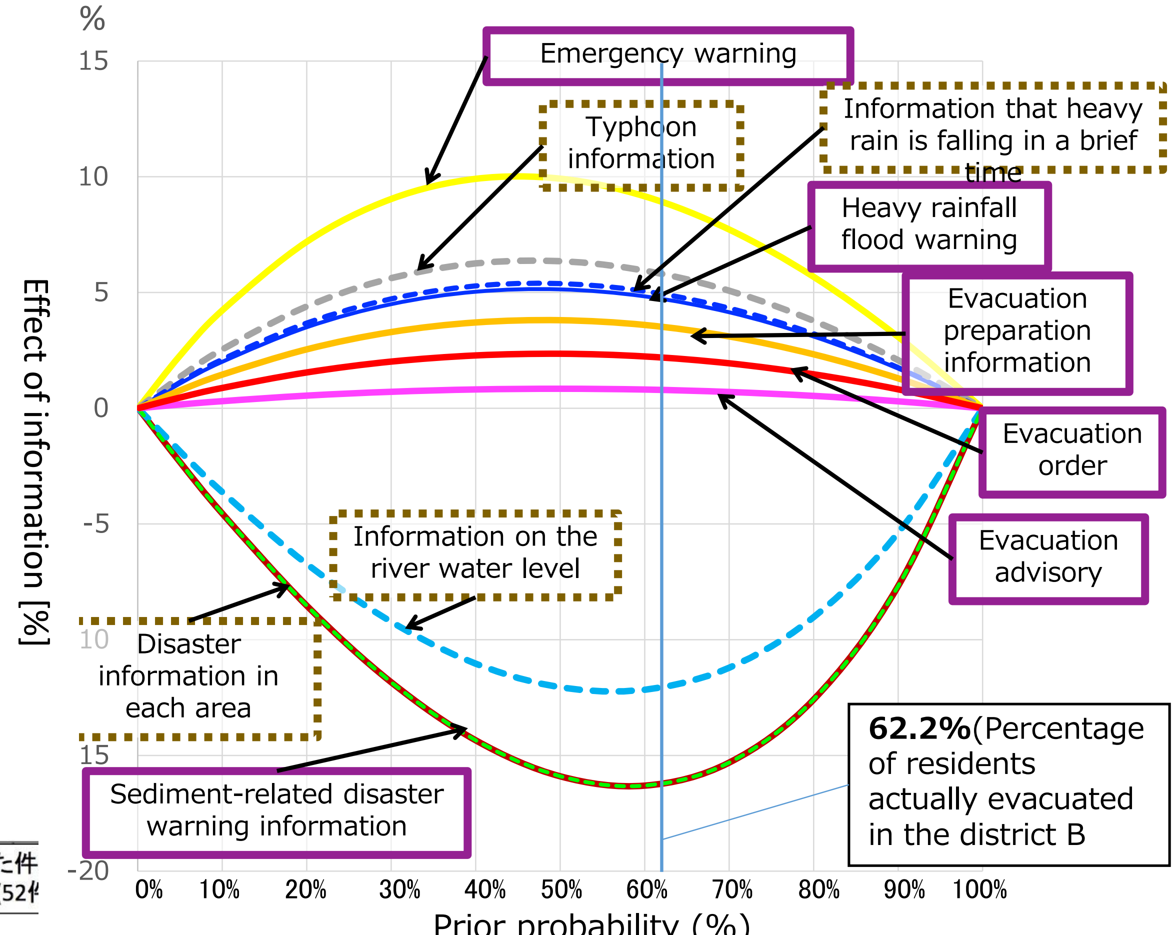


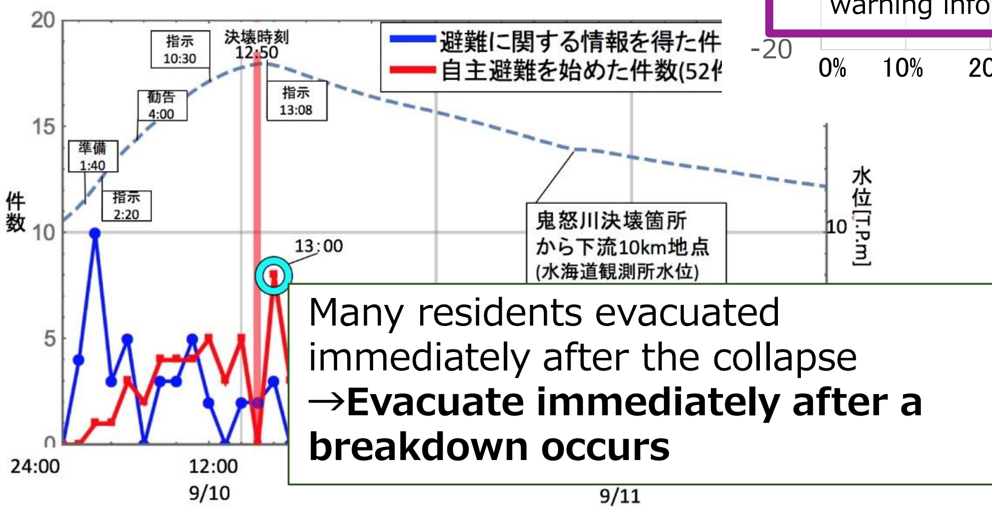
・Effective Information for evacuation has both probability information and fact information. Particularly, advance information about rainfall and river water level is effective for evacuation.

District B(Around the broken part of the embankment)



ヒアリング実施箇所の区分図

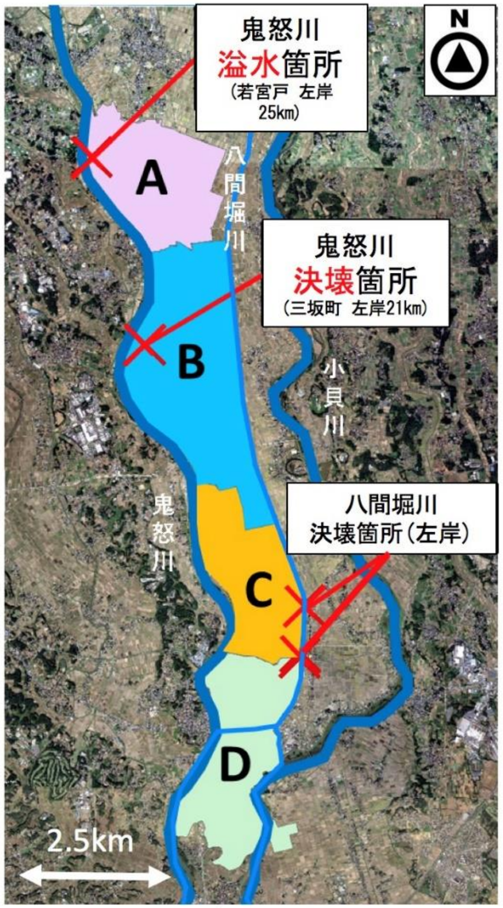




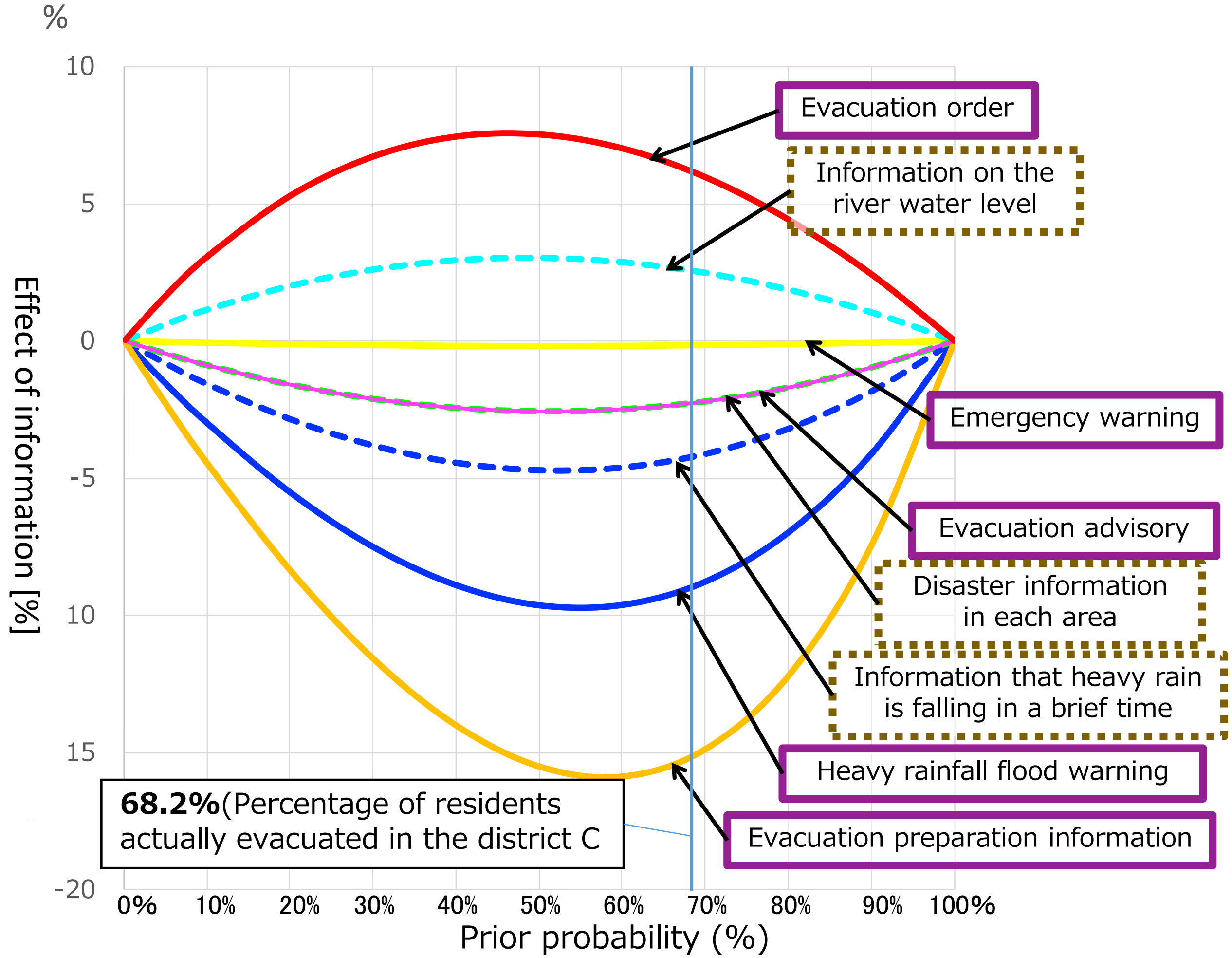
Many residents evacuated immediately after the collapse
→**Evacuate immediately after a breakdown occurs**

Information on evacuation (recommendation, instruction · evacuation effect of preparation information) is small effect
→There was no time delay to utilize since the evacuation direction was issued immediately after the collapse

District C(Between the broken part of the embankment and a city area)

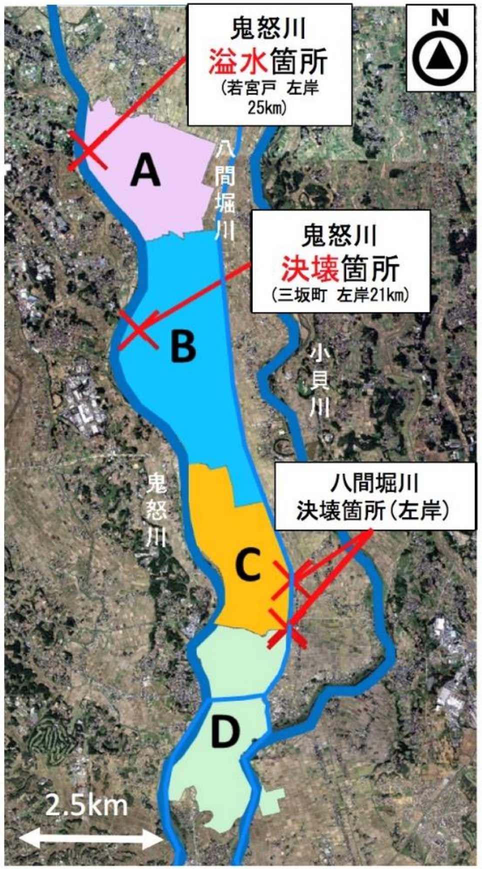


ヒアリング実施箇所の区分図

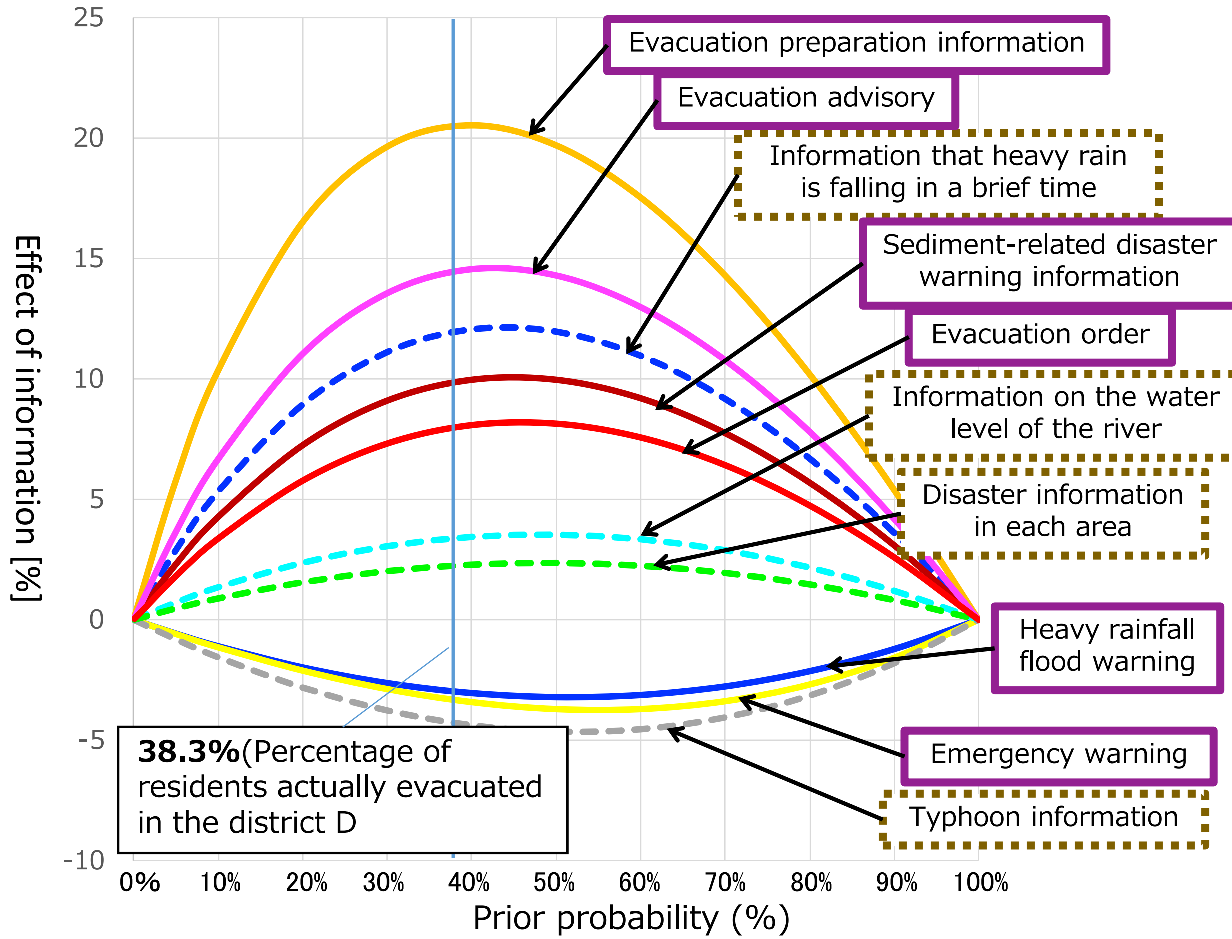


- There are few information effective for evacuation, information on evacuation instructions and river water level.

District D(around the city area)



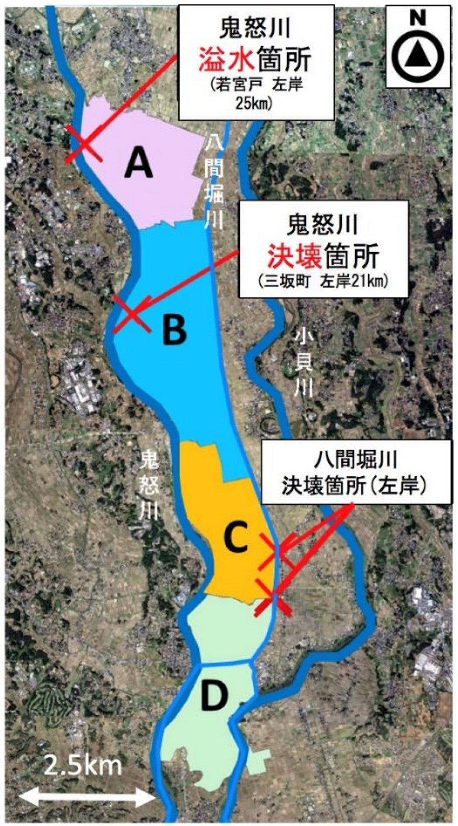
ヒアリング実施箇所の区分図



・Effective information for evacuation has both probability information and fact information

→ In particular, it was information on evacuation such as evacuation advisory and evacuation preparation information.

Summary of the disaster information and evacuation situation by district

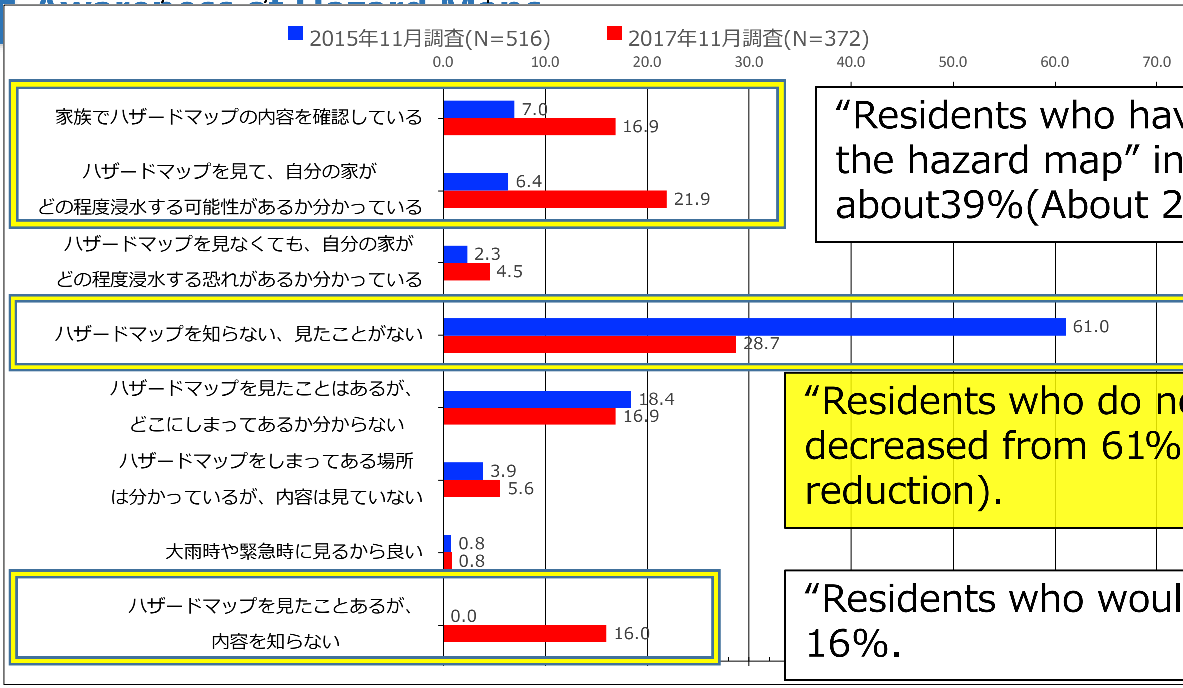


ヒアリング実施箇所の区分図

	District features	Information effective for evacuation (Using Bayes Theorem)
District A (around the overflow are)	Residents recognize the risk of flooding from daily and evacuate immediately after obtaining evacuation information. →Disaster prevention consciousness is high	In particular, advance information to be issued before the occurrence of the disaster of rainfall amount and river water level.
District B (Around the broken part area)	Difficult of embankment breakdown occurred. Many people evacuated immediately after the collapse.	Information on evacuation such as evacuation instructions has less effect on evacuation. Because the evacuation direction was the issuance immediately after the collapse, I could not afford at that time to make use of it.
District C (Between the broken part area)	Evacuation start is late.	Two less effective information. The effective is evacuation instructions and the water level of the river.
District D (A city area)	There are few people who evacuated.	In particular, information on evacuation such as evacuation recommendations and evacuation preparation information.

Awareness of Hazard Maps

Question：Have you seen your local hazard map?

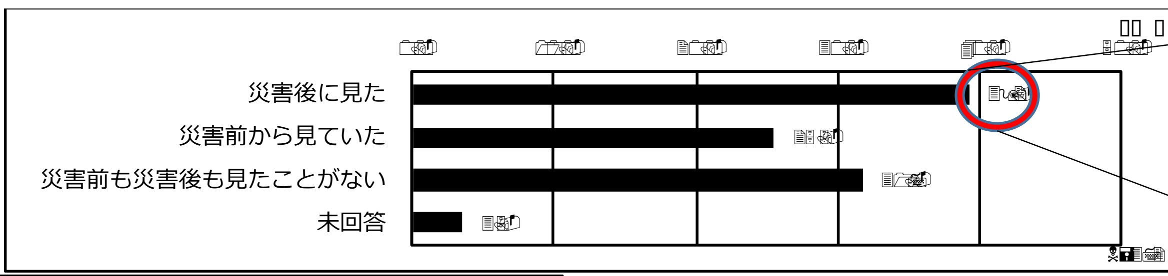


“Residents who have reviewed the contents of the hazard map” increased from about 13% to about39%(About 25% increase).

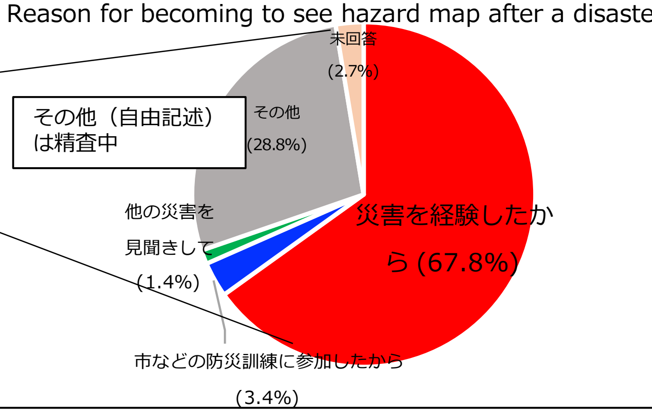
“Residents who do not know the hazard map at all” decreased from 61% to about 29%(About 30% reduction).

“Residents who would only read the hazard map” is 16%.

Question：Have you seen the hazard map after the disaster?



Question：Reason for becoming to see hazard map after a disaster

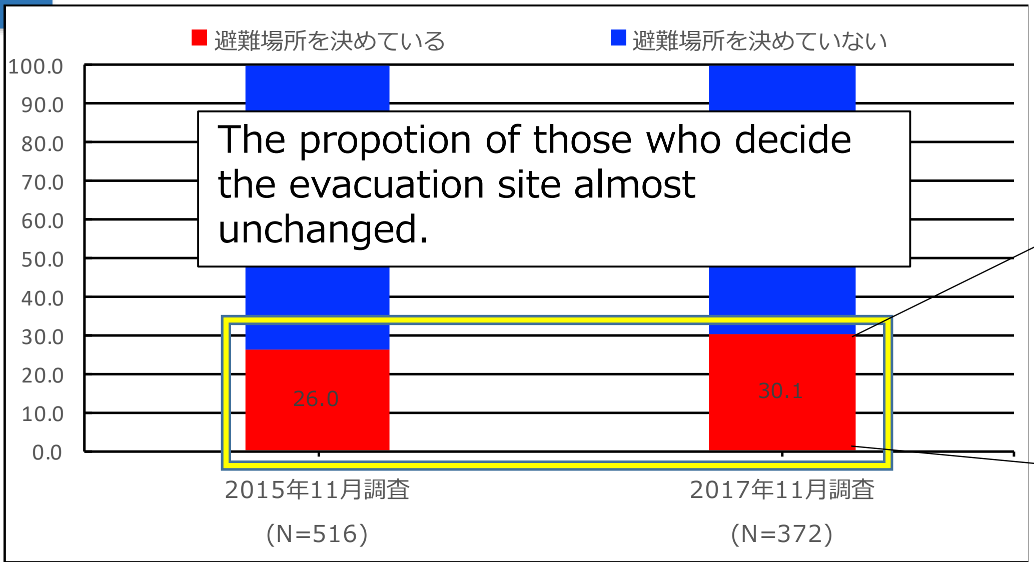


About 40% of the 65% people (Residents who saw a hazard map) answered “they saw a hazard map after disaster”.

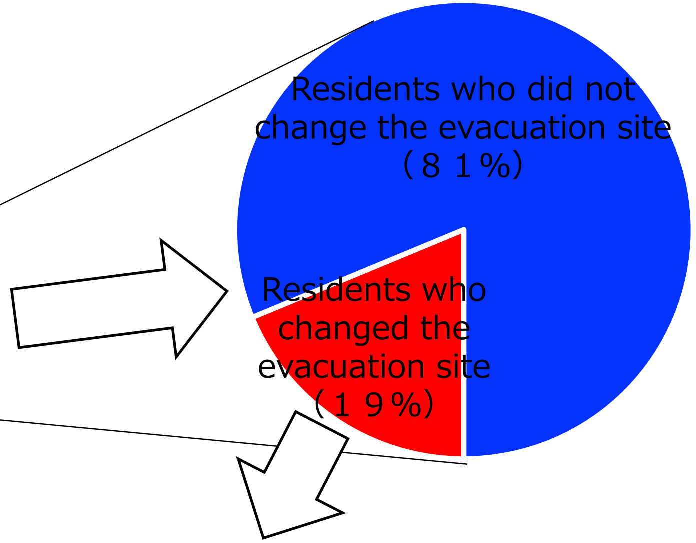
There were 68% who answered 「Residents who experienced a disaster」 was the largest. In Joso City, disaster drills such as “Because residents who participated in disaster drills such as municipalities” was as low as about 4%”.

Evacuation location decision

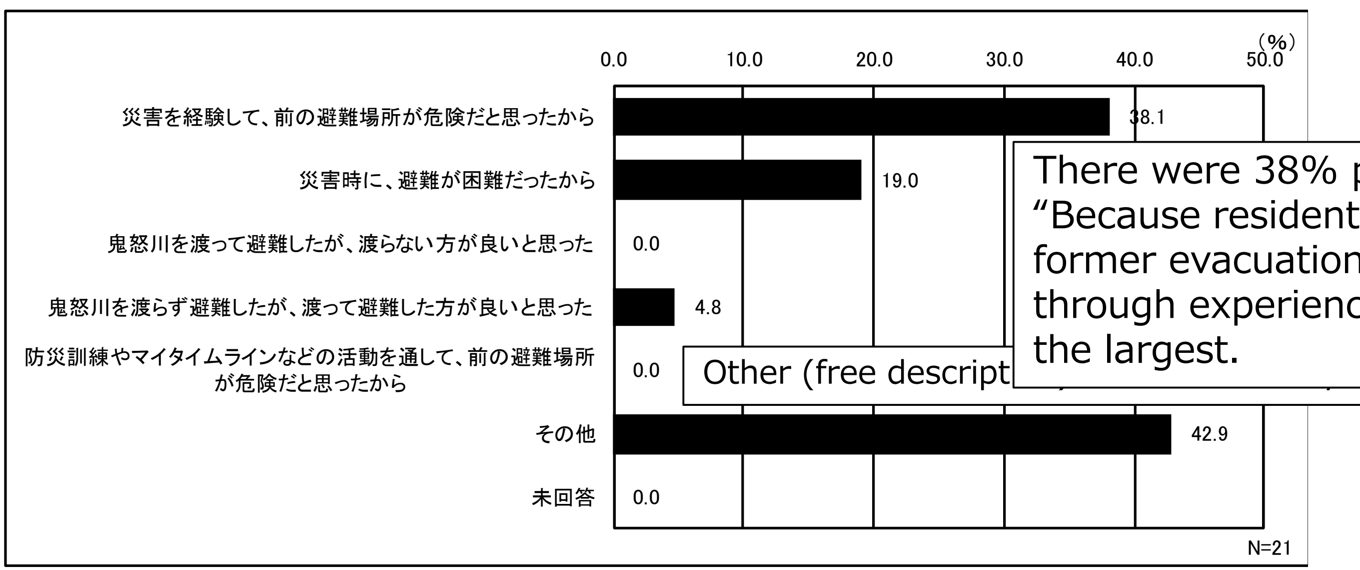
Question : Do you decide where to evacuate with your family?



Question : Did you change the evacuation site after the disaster?



Question : Why did you change the evacuation site after the disaster?



Reproduction of inundation situation in Joso City by flood inundation analysis

Analysis of river and flooplain integrated

Basic equation(shallow water equations)

$$\frac{\partial h}{\partial t}+\frac{\partial M}{\partial x}+\frac{\partial N}{\partial y}=0$$

$$\frac{\partial M}{\partial t}+\frac{\partial}{\partial x}\left(\frac{M^2}{h}\right)+\frac{\partial}{\partial y}\left(\frac{MN}{h}\right)=-gh\frac{\partial H}{\partial x}-\frac{gn^2u\sqrt{u^2+v^2}}{h^{1/3}}$$

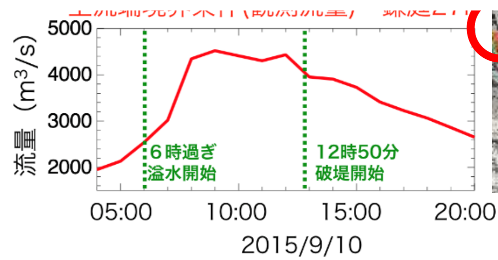
$$\frac{\partial N}{\partial t}+\frac{\partial}{\partial x}\left(\frac{MN}{h}\right)+\frac{\partial}{\partial y}\left(\frac{N^2}{h}\right)=-gh\frac{\partial H}{\partial y}-\frac{gn^2v\sqrt{u^2+v^2}}{h^{1/3}}$$

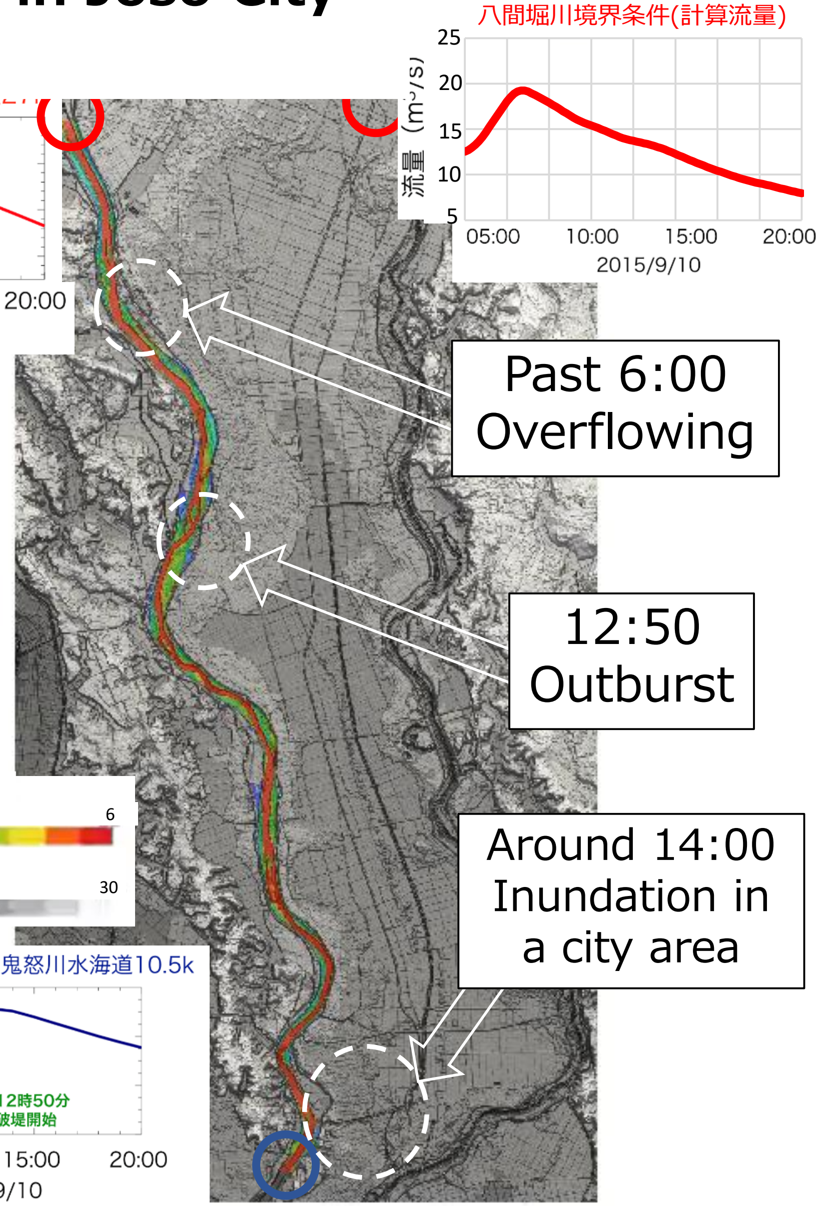
M, N : Discharge flux in x and y direction
t : Time coordinates、 x, y : plane coordinates
h : Water depth、 g : gravitational acceleration
n : roughness length、 H : water level
u, v : flow verocity in x and y directions

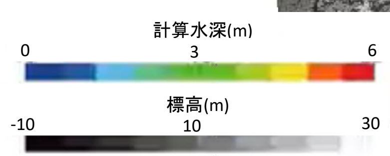
Differentiated by a Leap-frog algorithm

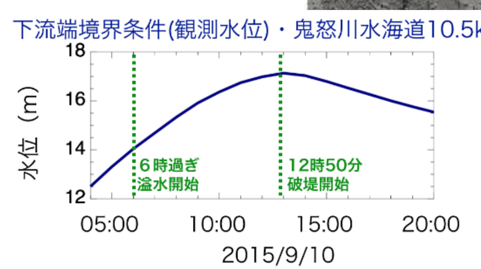
$\Delta x=\Delta y=10\text{m}$、 $\Delta t=0.2\text{s}$

Long calculation time !
(It takes one day to reproduce the data for one day)









2015/9/10 04:00

Inundation flow analysis by Topography Fitting Grid Model

Basic equation(shallow water equations)

$$\frac{\partial h}{\partial t} + \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} = 0$$

$$\frac{\partial M}{\partial t} + \frac{\partial}{\partial x}\left(\frac{M^2}{h}\right) + \frac{\partial}{\partial y}\left(\frac{MN}{h}\right) = -gh\frac{\partial H}{\partial x} - \frac{gn^2u\sqrt{u^2+v^2}}{h^{1/3}}$$

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}\left(\frac{MN}{h}\right) + \frac{\partial}{\partial y}\left(\frac{N^2}{h}\right) = -gh\frac{\partial H}{\partial y} - \frac{gn^2v\sqrt{u^2+v^2}}{h^{1/3}}$$

Ignore the advection term

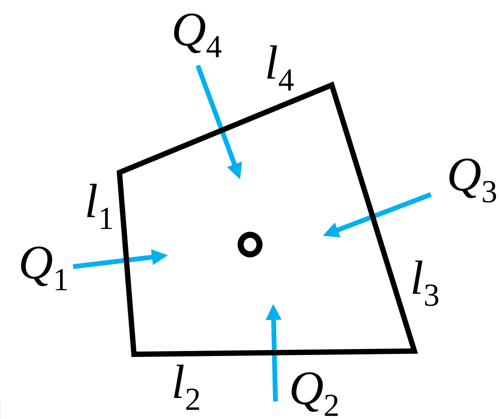
M, N : discharge flux in x and y direction
t : time coordinates、 x, y : plane coordinates
h : water depth、 g : gravitational acceleration
n : roughness length、 H : water level
u, v : flow verocity in x and y directions

Extend to linear flooding model that can be calculated using topography-fitting grid(Yasuda・Yamada*)

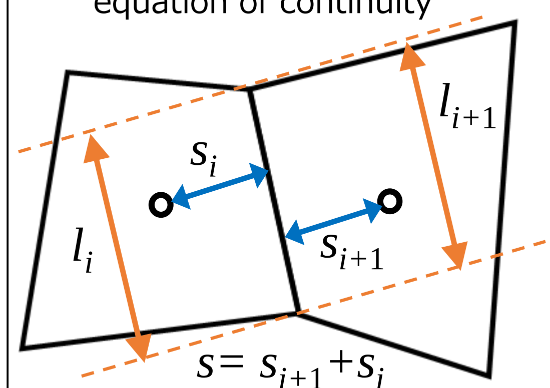
$$\frac{\partial \eta}{\partial t} = \frac{1}{A}\left(\sum_{i=1}^{N} Q_i\right)$$

$$\frac{\partial Q}{\partial t} + ghl\frac{\partial \eta}{\partial s} = -\frac{gn^2|Q|Q}{h^{7/3}l}$$

η : water level of flood、 h : water depth 、
t : time coordinates 、
s : plane coordinates (distance of center of figure between adjacent grids)、
A : grid area
g : gravitational acceleration
n : roughness length 、
Q_i : inflow from adjacent grid、
N : total number of edge of grid i 、
l : length of edge of grid



Variable definition of equation of continuity



Variable definition of equation of motion

Inundation flow analysis by Topography Fitting Grid Model

$$\frac{\partial \eta}{\partial t} = \frac{1}{A}\left(\sum_{i=1}^{N} Q_i\right)$$

$$\frac{\partial Q}{\partial t} + ghl\frac{\partial \eta}{\partial s} = -\frac{gn^2|Q|Q}{h^{7/3}l}$$

η : water level of flood、 h : water depth 、
t : time coordinates 、
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(distance of center of figure between adjacent grids)、
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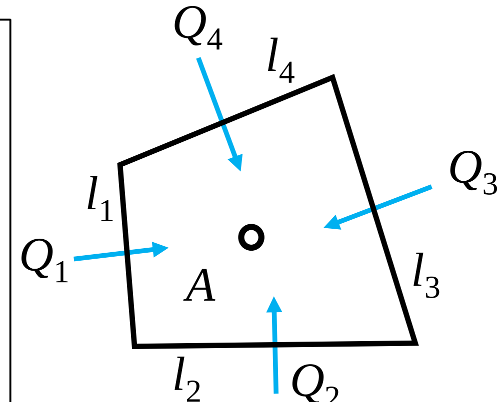
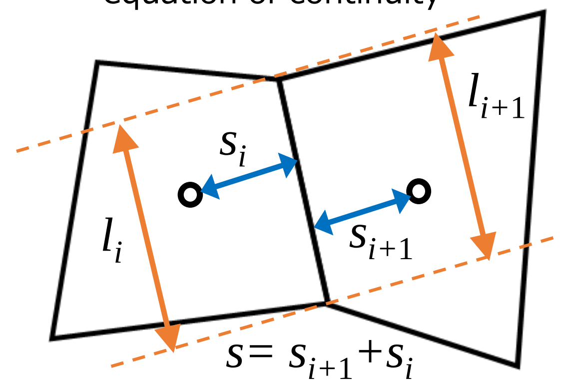
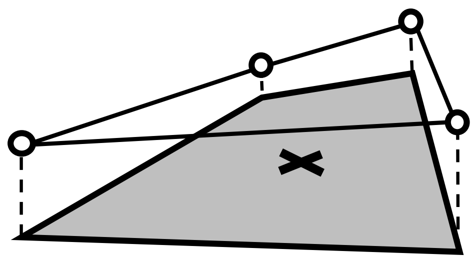


Variable definition of equation of continuity

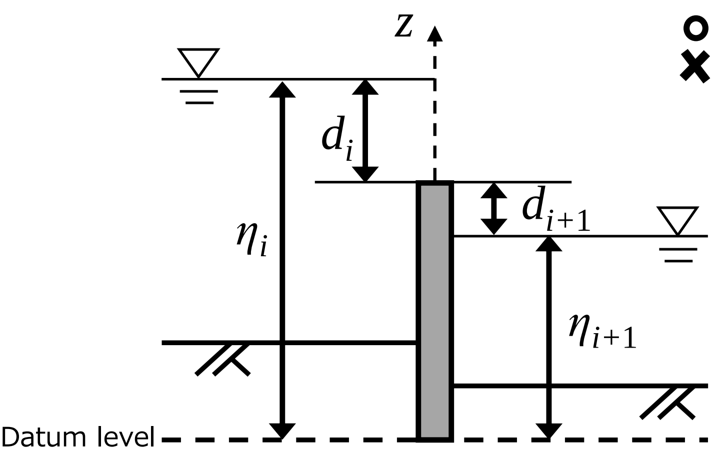


Variable definition of equation of motion

About linear boundary



○ Altitude of linear boundary
✕ Average altitude of the ground in the grid

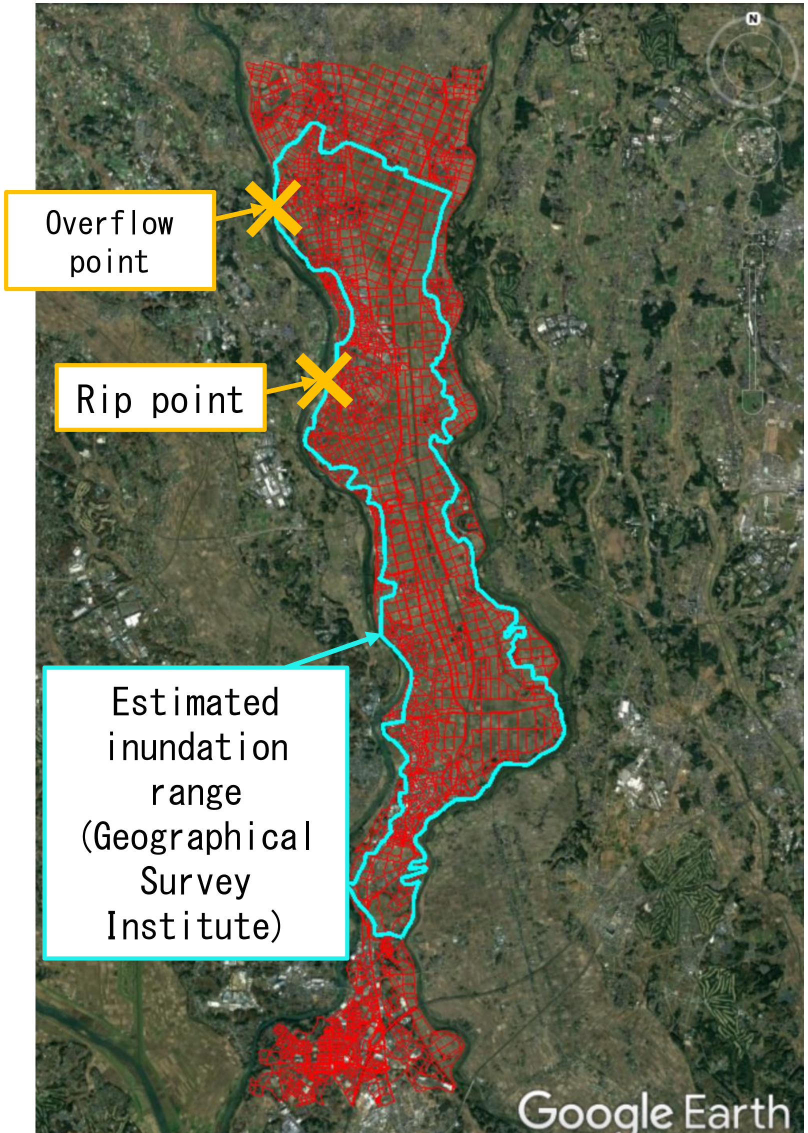


Linear boundary

	$d_{i+1} > 0$	$d_{i+1} \le 0$
$d_i > 0$	$h_{i+1/2} = \frac{d_i + d_{i+1}}{2}$	$h_{i+1/2} = \frac{d_i}{2}$
$d_i \le 0$	$h_{i+1/2} = \frac{d_{i+1}}{2}$	$Q = 0$

Inundation flow analysis by Topography Fitting Grid Model

grid division

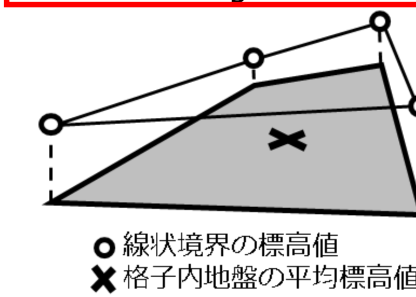


It is shown by Fukuoka and others (1994,1998) and Inoue, Toda and others that it is necessary to divide a road and the ridge into a case to ① lane ② obstacle to the spread of the flooding water by a pitch difference with width and neighboring ground height in flooding analysis.

① The example that a road plays a role as the lane

② The example that a road obstacles to the spread of the flooding water

A road and the ridge assume it a linear border in defiance of width



We divided an analysis domain into a lattice by road centerline shown in OpenStreetMap (free database)

Inundation flow analysis by Topography Fitting Grid Model

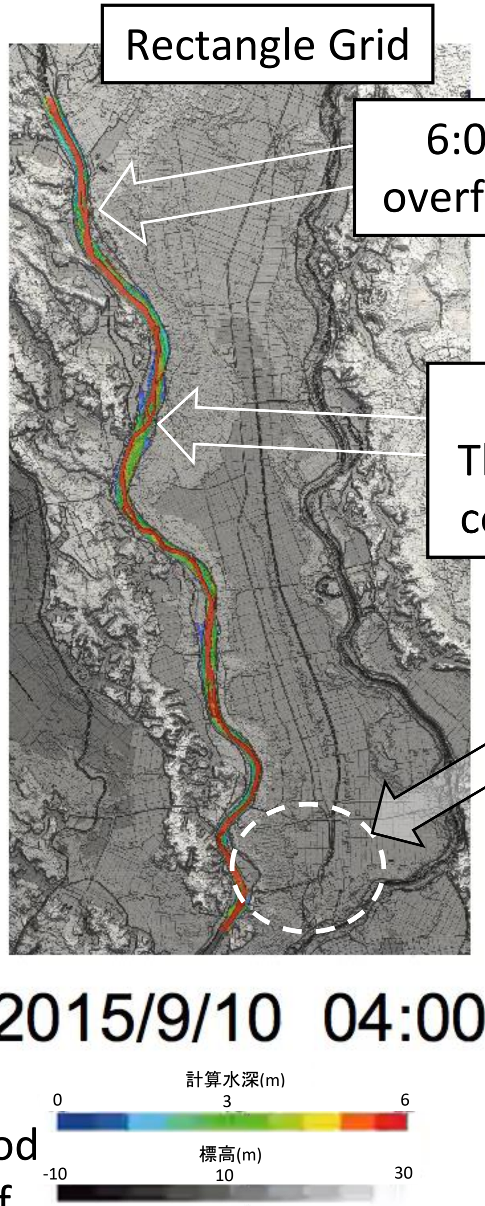
Term
9/10 4:00~20:00

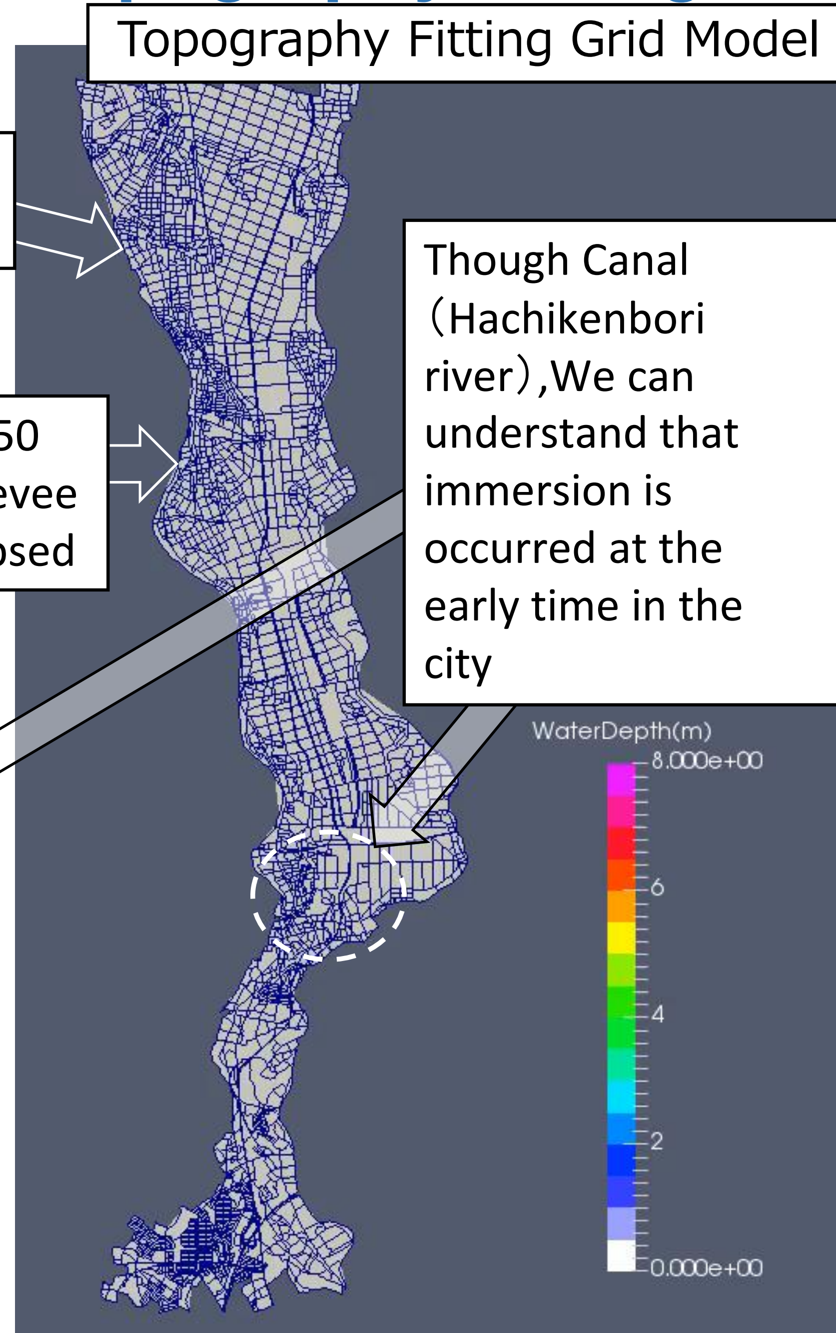
Rectangle Grid
Number of grid : 1232065
Grid size : 10m × 10m
Calculation time : about 24 hour
（about 1440 minutes）

grid adapting terrain
Number of grid : 3337
Calculation time: About 10 minutes

144 times faster!

Real time forecast of flood
+ Apply the simulation of the evacuation behavior



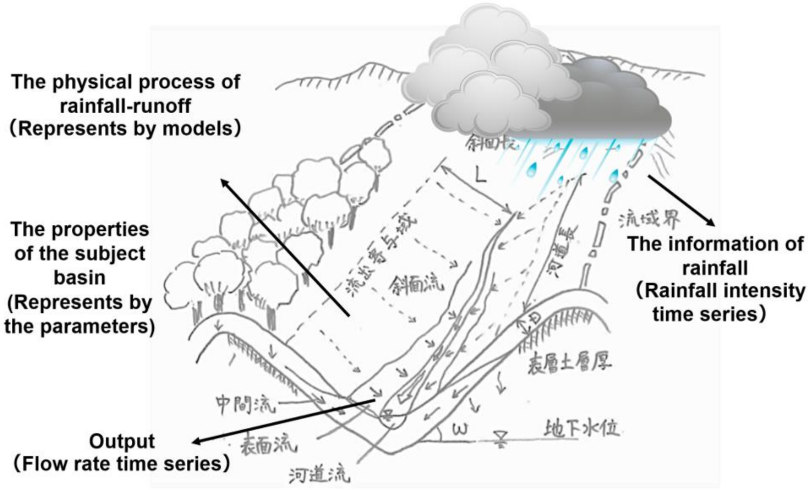


Chapter 3

Rainfall-runoff analysis considered the uncertainty of rainfall based on Ito's stochastic differential equation theory

Introduction

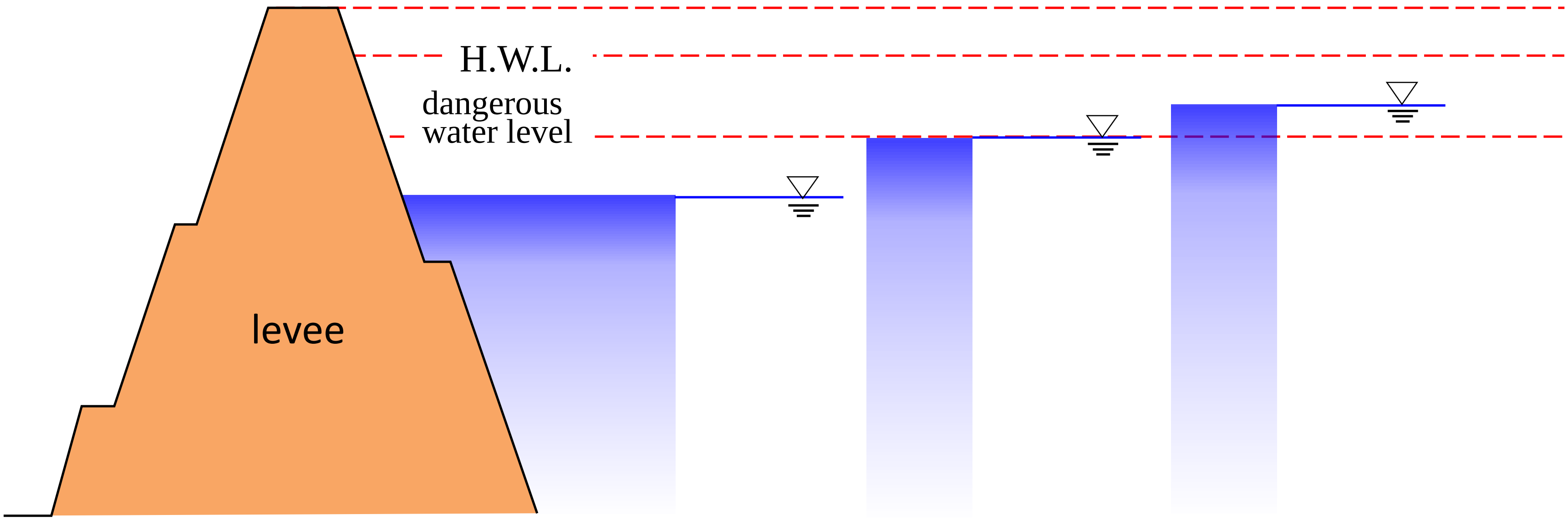
Serious flood disasters in Japan



Basic concept of rainfall-runoff analysis

Modelized the basin, think the rainfall as input, and then we can get the time evolution of the water level.

According to the result, government can give warnings to the citizens.

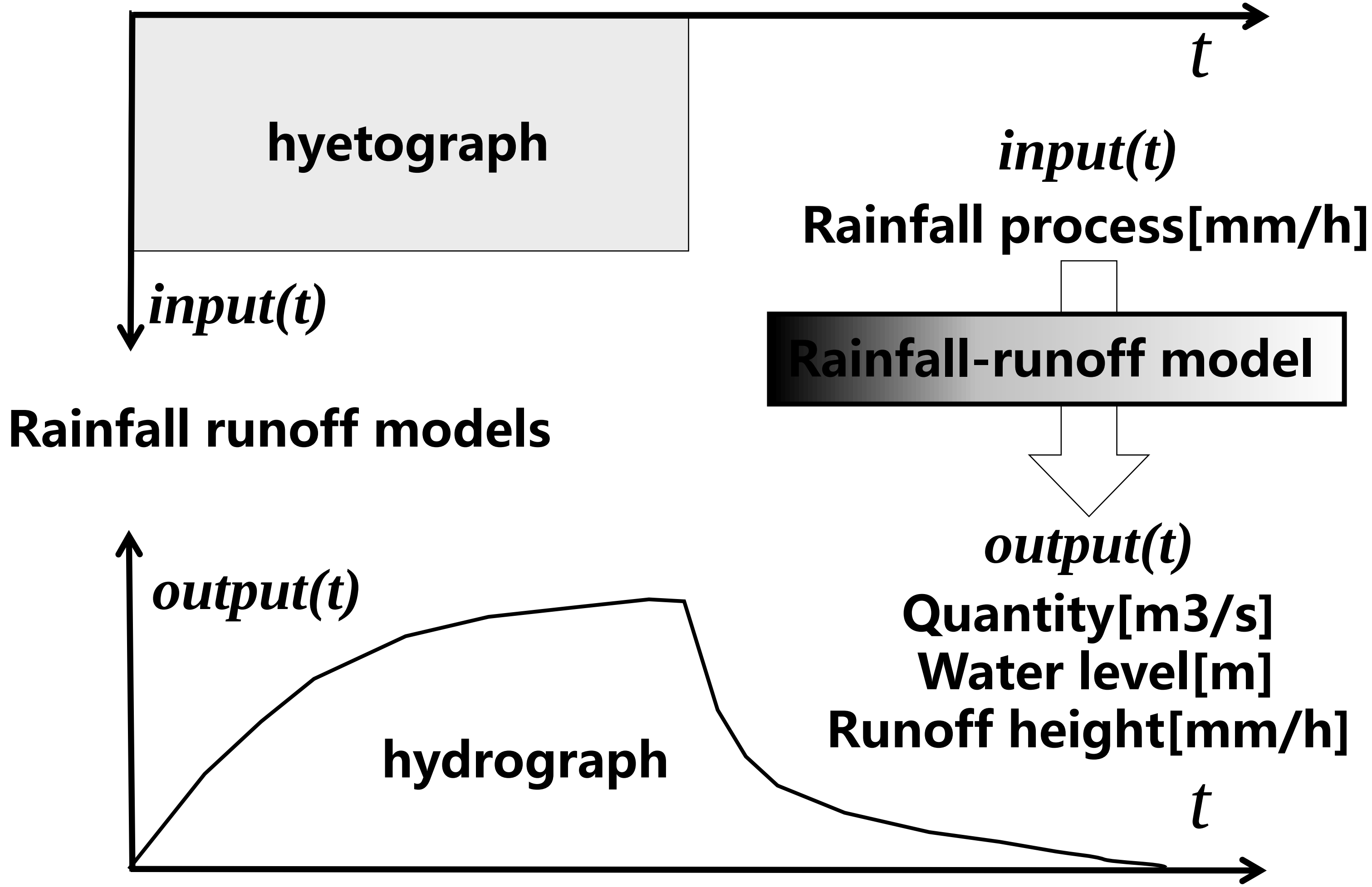


High water level(H.W.L.) : The most important index in flood control which considered as the design external force of levee. This index is calculated by the theory of extreme value statistic using historical hydrology data.

Flood monitoring and forecasting : After H.W.L. had been designed, The levee will be designed strong enough to resist the H.W.L., so, it is very important monitor and forecast the water level in a flood event. By compare the water level to H.W.L.(or other evaluation index like dangerous water level), we can know how

Deterministic rainfall-runoff models

A brief description about rainfall-runoff problem



Modeling of rainfall-runoff system

Deterministic rainfall-runoff models

The basic equation of rainfall-runoff process for simple slope

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=r_e(t)$$

$$v=\alpha h^m,\quad q=vh=\alpha h^{m+1}$$

Concept of the simple slope model

Using the continuous equation and the momentum equation we can get:

$$\frac{\partial q(x,t)}{\partial t}+(m+1)\alpha^{\frac{1}{m+1}}q(x,t)^{\beta}\frac{\partial q(x,t)}{\partial x}=(m+1)\alpha^{\frac{1}{m+1}}q(x,t)^{\beta}r_e(t)$$

Lumped

Assuming that the direct outflow will only take place near the river channel, so the outflow will be in in proportion to the length of slope

$$q(x,t)\cong xq_*(t)$$

The outflow take place at *x*=*L*

$$\frac{dq_*}{dt}=a_0q_*^{\beta}\left(r_e(t)-q_*\right)$$

$\alpha=\frac{k_s i}{D^{\gamma-1}w^{\gamma}}$ $\beta=\frac{m}{m+1}$

$a_0=\frac{\beta}{1-\beta}\left(\frac{\alpha}{L}\right)^{1-\beta}$

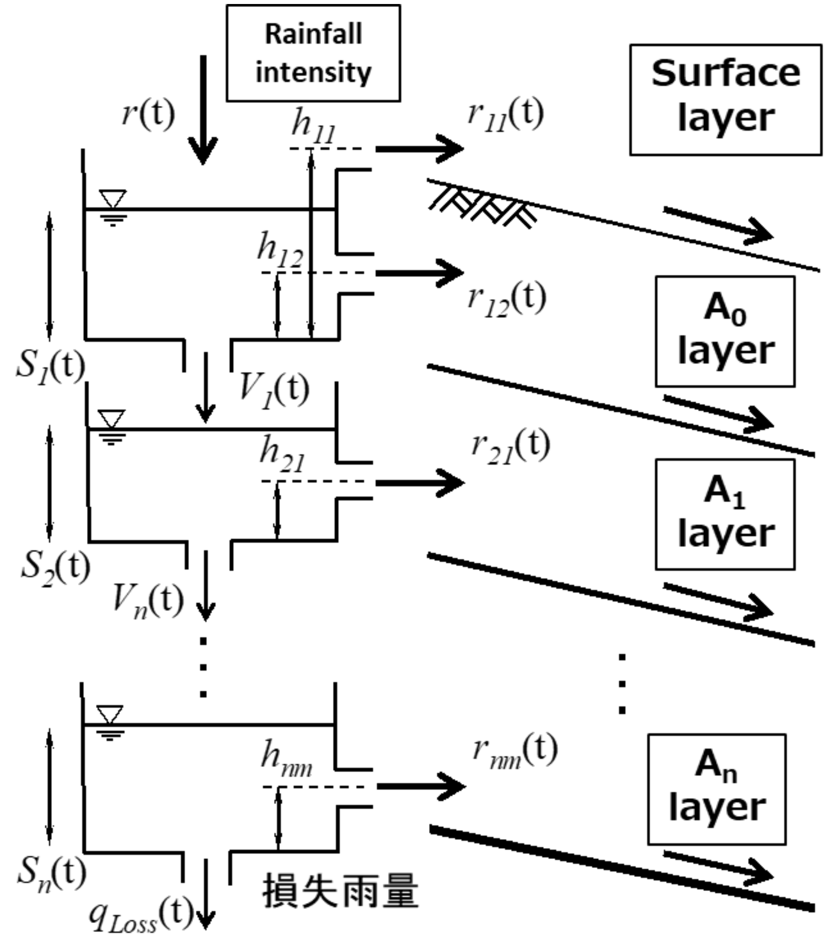
v:Average velocity of cross section[mm/h],
h:Submerged depth [mm]
q_* :Flow rate[mm/h] α,m:Parameters

Deterministic rainfall-runoff models

Expand the model to multi-layers model

$$
\begin{cases}
\dfrac{dq_{nm}}{dt} = \alpha_{nm} q_{nm}^{\beta_{nm}} (r_{nm} - q_{nm}) \\
\dfrac{ds_n}{dt} = V_{n-1} - r_{nm} - V_n \\
\begin{cases} r_{nm} = 0 & (s_n < h_{nm}) \\ r_{nm} = a_{nm}(s_n - h_{nm}) & (s_n \geq h_{nm}) \end{cases} \\
q_{Loss} = V_n = b_n s_n
\end{cases}
$$

n : Layer index
m : runoff index for each layer



According to Yoshimi, Yamada's research, the basic equation for simple slope can be expand to multi layers. By doing so, the model can deal with basins with multi layer soil structure and consider the vertical flow between this layers.

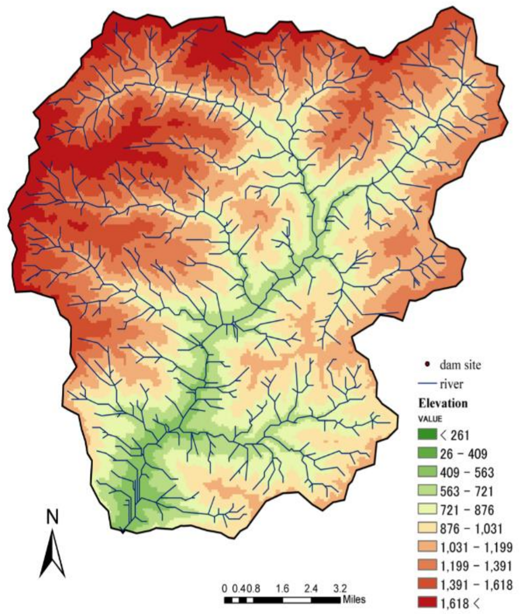
Basic equation for each layer:

$$\frac{dq_{nm}(t)}{dt} = a_{nm} q_{nm}(t)^{b_{nm}} (r_{nm} - q_{nm}(t))$$

Expand the rainfall-runoff model to multi-layers

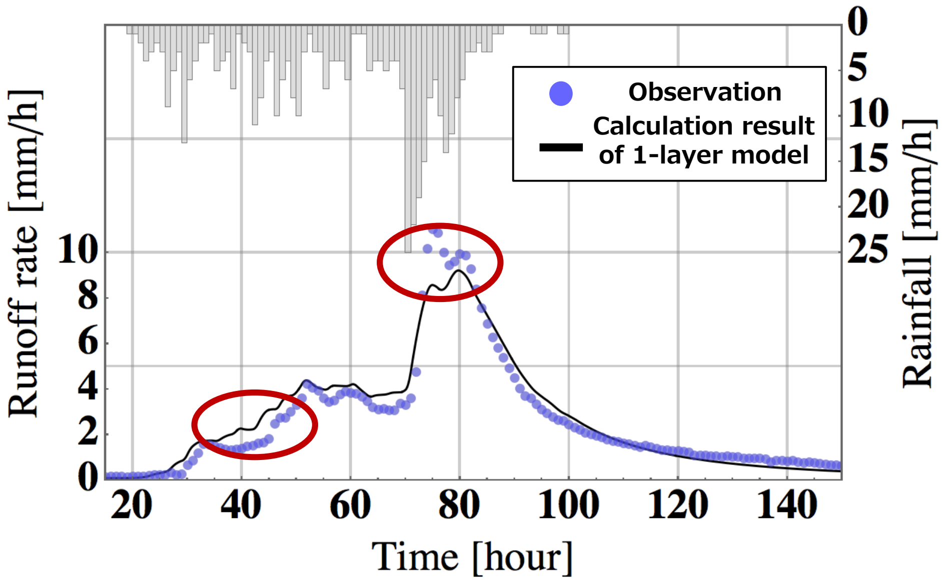
Deterministic rainfall-runoff models

Practical use of the basic equation for simple slope(Case study in Kusaki dam basin)



Kusaki dam basin

Parameters	Caption	Values
q_0 [mm/h]	Initial condition of the runoff height	0.1
D [mm]	Thickness of the surface soil layer	200
L [mm]	Length of modelized Slope	30000
k_s [mm/h]	Permeation coefficient of soil	360
w	Effective void ratio	0.42
m	Non dimensional parameter represents the resistance of soil	0.667
i	Gradient of slope	0.174

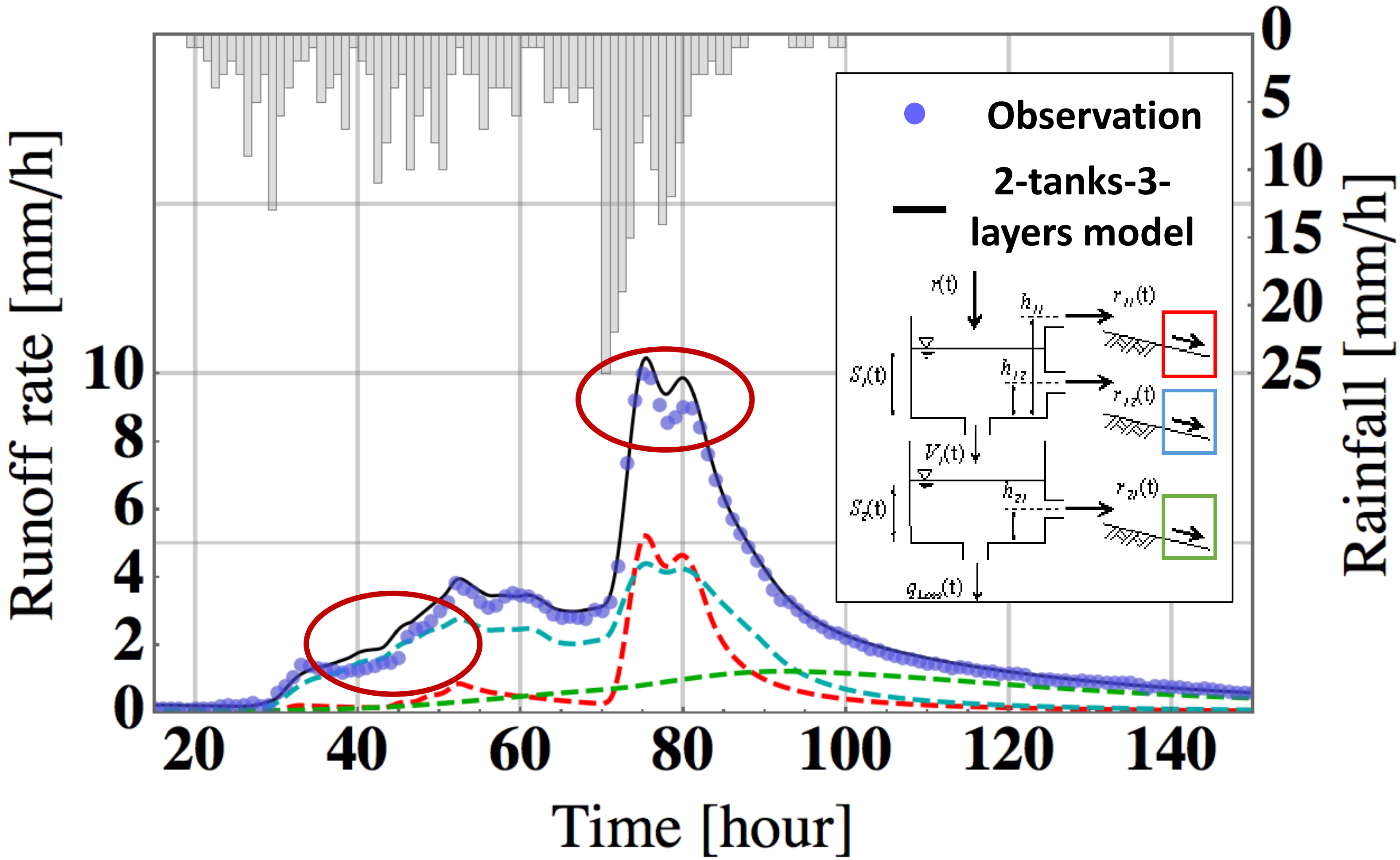


Simulation result of 1983-08-14 rainfall event in Kusaki dam basin

○ Using 1-layer model, the general shape of the runoff series is matching the observation series.

○ However, the rising part and peak of the runoff series is not quite matching the observation series.

Practical use of the 2-tanks-3-layers model(Case study in Kusaki dam basin)

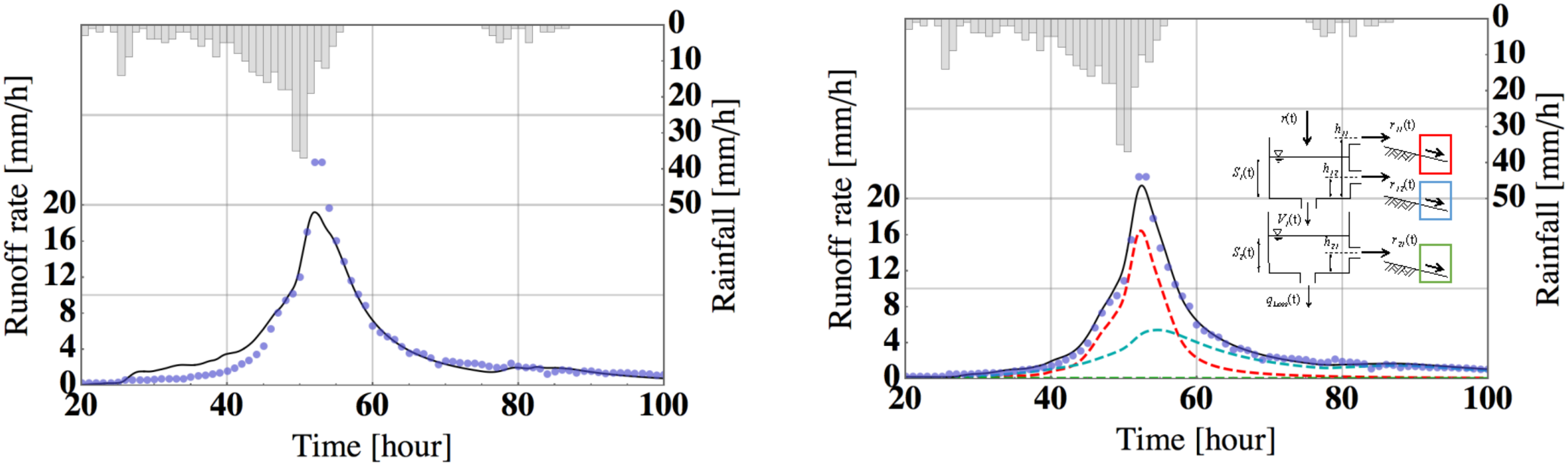


Simulation result of 1983-08-14 rainfall event in Kusaki dam basin

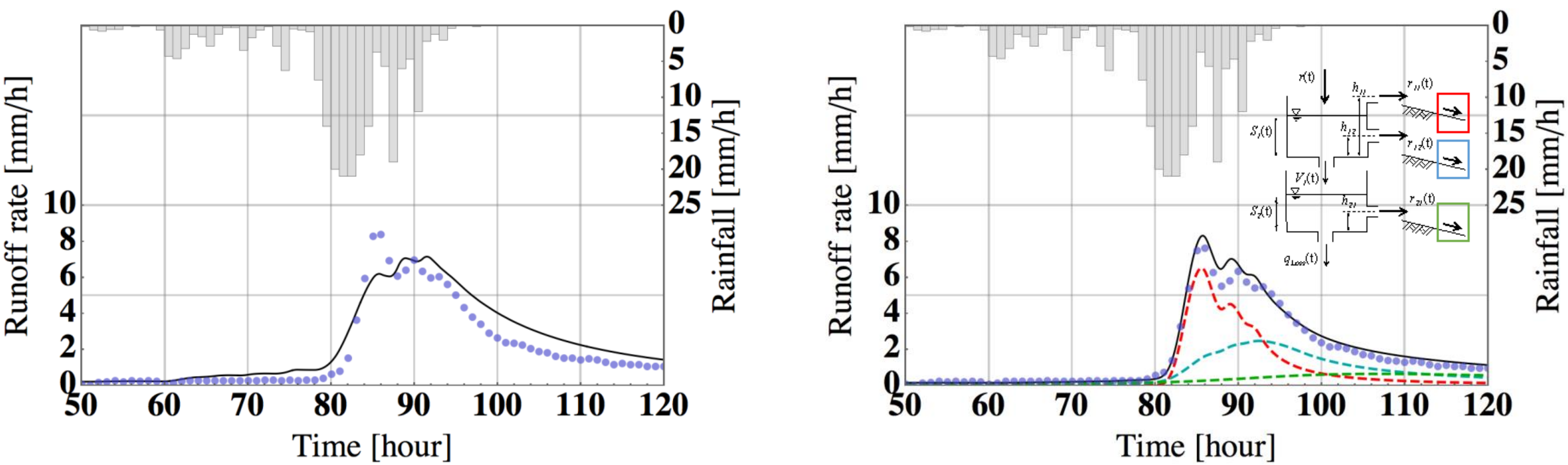
By compare the results of 1-layer model and 2-tanks-3-layers model, we can tell that the result of 2-tanks-3-layers matches the rising part and peak of the runoff series better.

Deterministic rainfall-runoff models

Compare the 2-tanks-3-layers model to 1-layer model



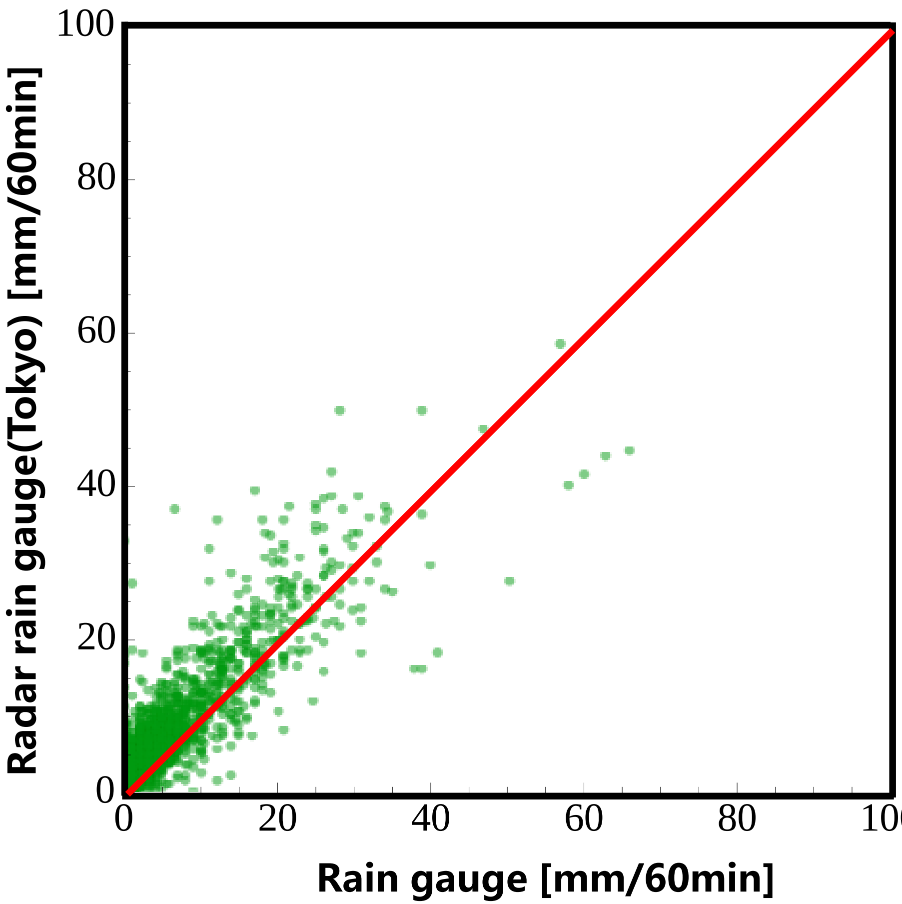
Simulation result of 1982-07-31 rainfall event in Kusaki dam basin



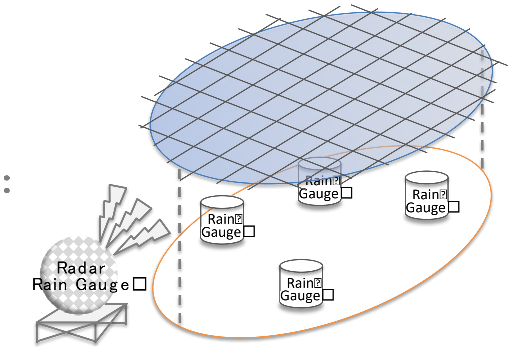
Simulation result of 1989-08-24 rainfall event in Kusaki dam basin

Uncertainty of rainfall intensity

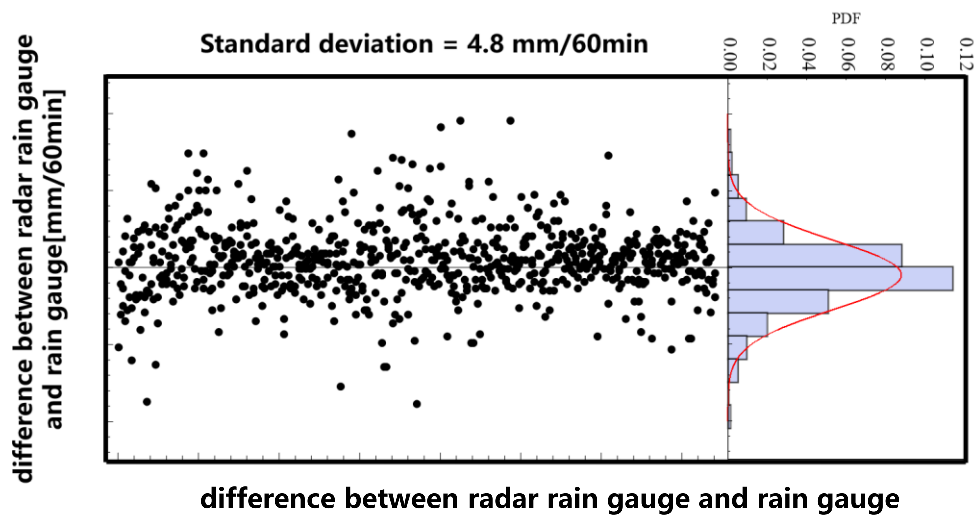
(Temporal & spatial distribution)



XRAIN
Temporal Resolution:
1 minute
Spatial Resolution:
250m×250m

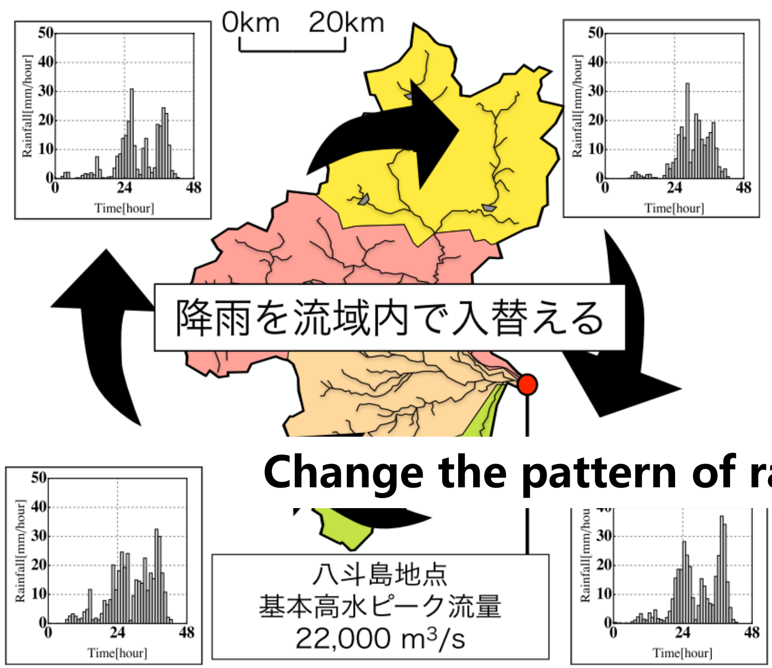


Radar(XRAIN) and Ground rain gauge

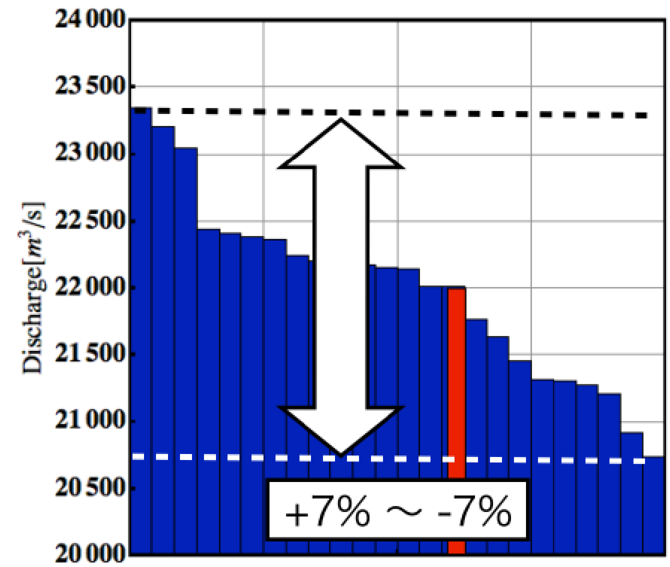


There is always a difference between the measurement of the rain gauge and the radar rain gauge system and there is no way to tell which one is the " true" rainfall.

Uncertainty of rainfall intensity
(Temporal & spatial distribution)



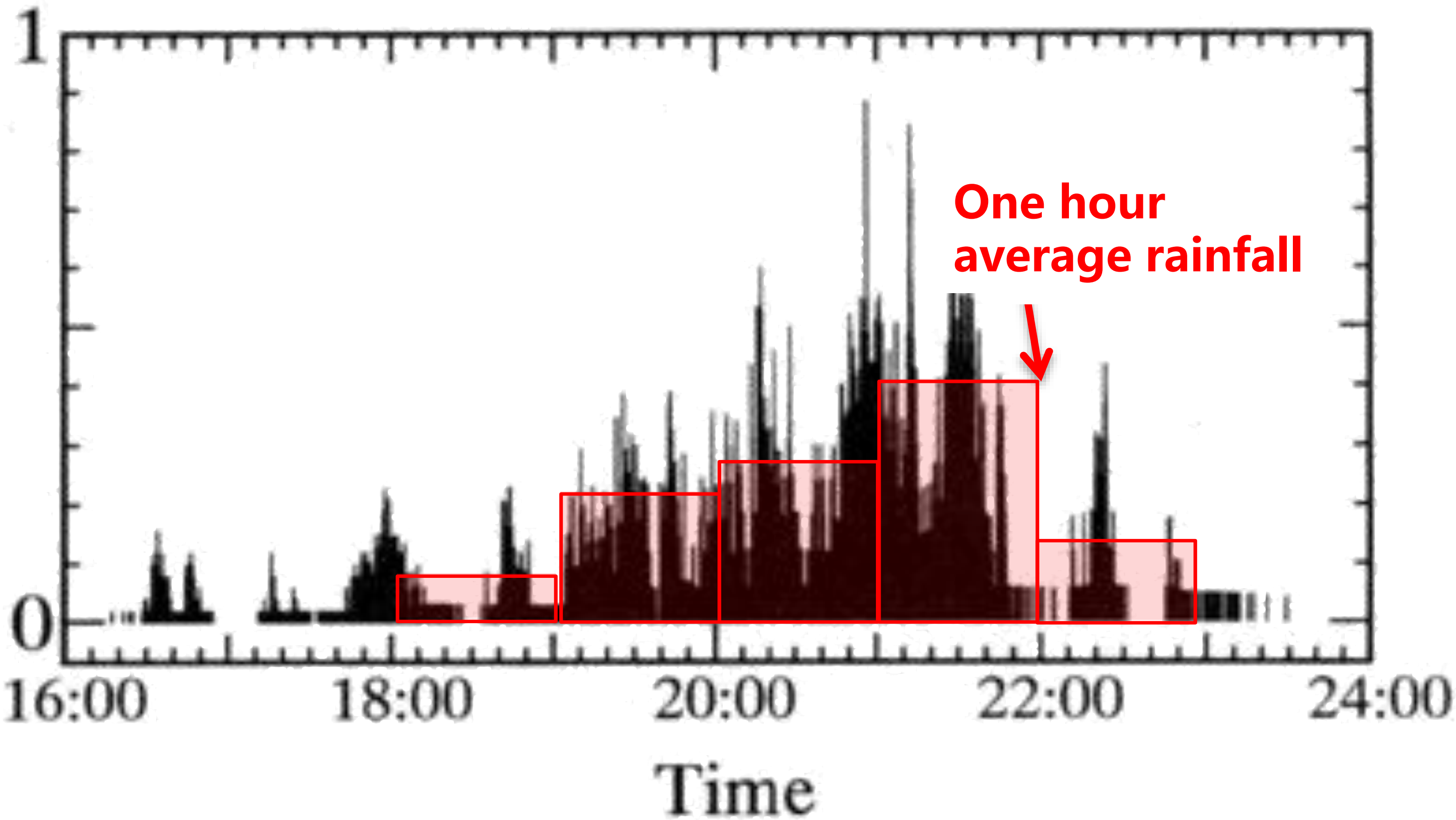
Change the pattern of rainfall



The peak discharge of Yattajima station is 22,000m³/s

This is a reproduce calculation of the typhoon Kathleen 1947 flood event in Tonegawa catchment area. Changing the pattern of rainfall between sub catchments can cause a difference of ±7% in peak discharge.

Uncertainty of rainfall intensity
(Temporal & spatial distribution)

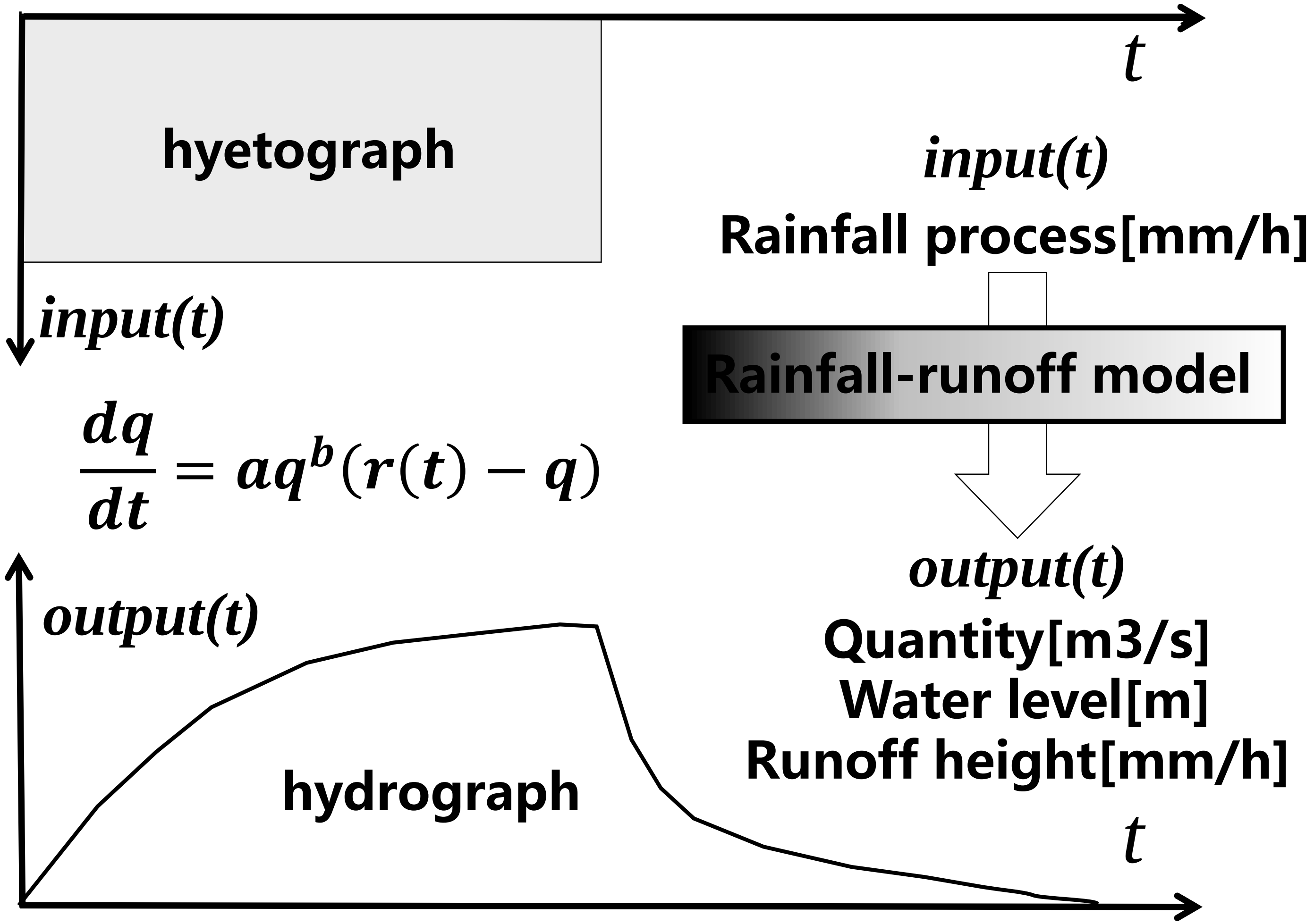


Data of laser rainfall(raindrop) rain gauge system developed by Yamada(1994)

It implies that one way to look at the rainfall intensity time series is to consider the average part as the deterministic part and the rest as stochastic part.

Deterministic rainfall-runoff models

Deterministic models cannot consider the uncertainty of rainfall-runoff process

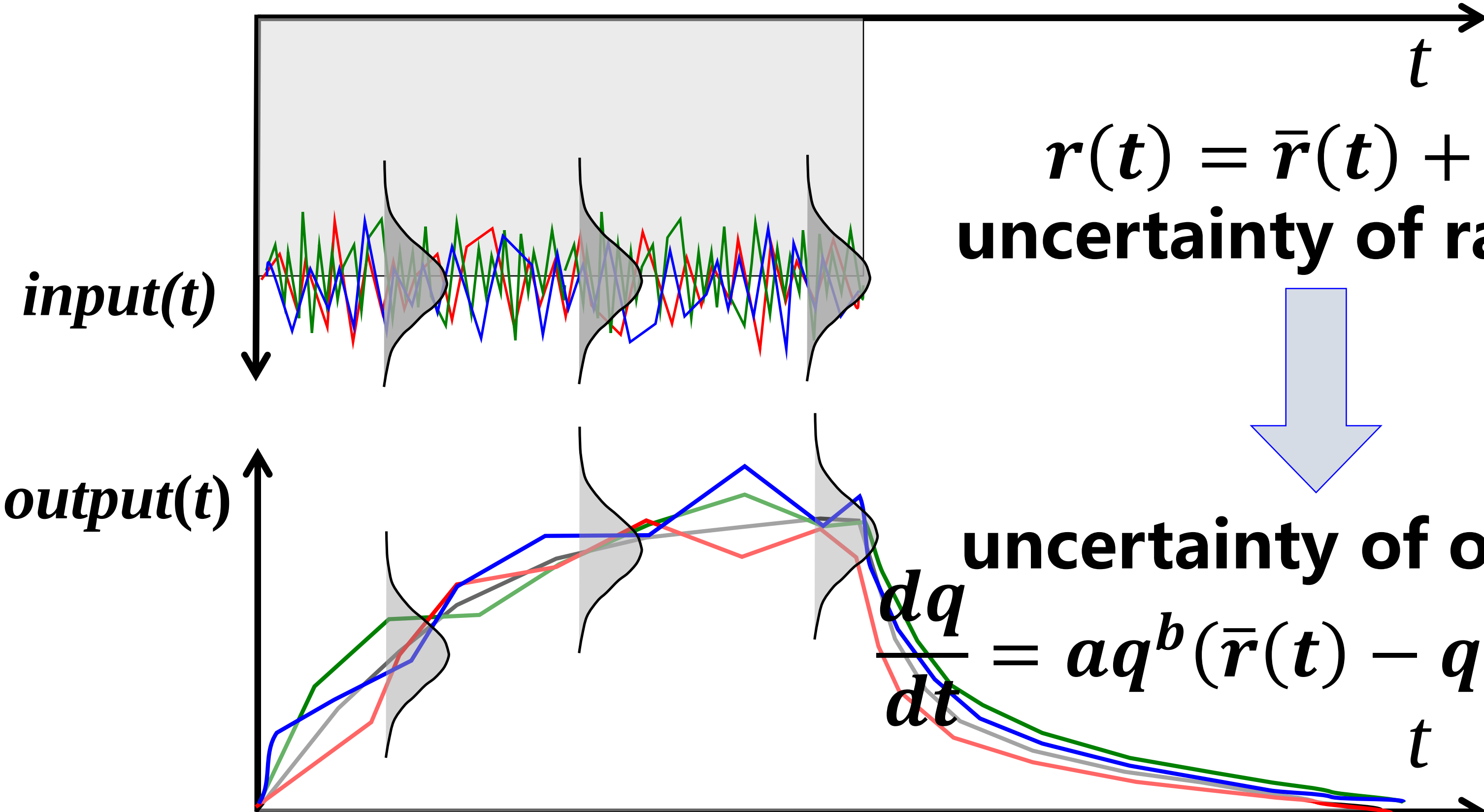


$$\frac{dq}{dt} = aq^b(r(t) - q)$$

Modeling of rainfall-runoff system

How to consider the uncertainty of rainfall

Using stochastic differential equation



$$r(t) = \bar{r}(t) + r'$$

uncertainty of rainfall

uncertainty of outflow

$$\frac{dq}{dt} = aq^b(\bar{r}(t) - q) + aq^b r'$$

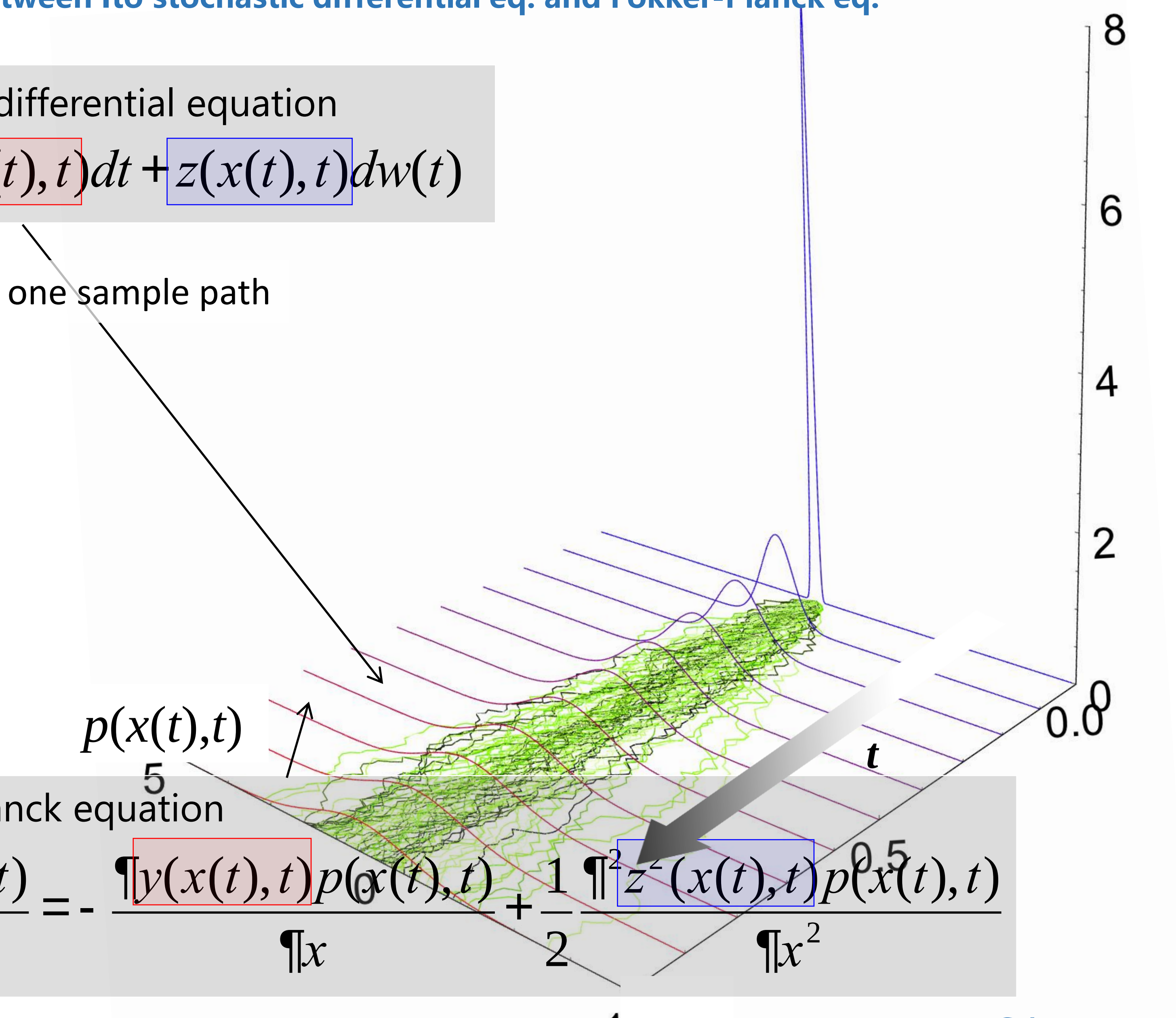
We want to know uncertainty of runoff caused by uncertainty of rainfall.

Physical systems with random external force

The relation between Ito stochastic differential eq. and Fokker-Planck eq.

Ito Stochastic differential equation

$$dx(t) = y(x(t),t)\,dt + z(x(t),t)\,dw(t)$$



Fokker-Planck equation

$$\frac{\partial p(x(t),t)}{\partial t} = -\frac{\partial y(x(t),t)\,p(x(t),t)}{\partial x} + \frac{1}{2}\frac{\partial^2 z^2(x(t),t)\,p(x(t),t)}{\partial x^2}$$

Background

Runoff analysis introducing stochastic process theory

From Ito's stochastic differential equation to Fokker-Planck equation

$$dX(t) = y(X(t),t)dt + \sigma(X(t),t)dw$$

$$(dX)^2 = y(X(t),t)^2(dt)^2 + 2y(X(t),t)\sigma(X(t),t)dtdw + \sigma(X(t),t)^2(dw)^2$$

$$(dX)^2 = y(X(t),t)^2(dt)^2 + 2y(X(t),t)\sigma(X(t),t)dtdw + \sigma(X(t),t)^2 dt$$

$(dw)^2 = dt$

Using the property of Winnier process

$$\frac{d}{dt}E\left(h(X(t))\right) = E\left(\frac{d}{dt}h(X(t))\right)$$

$$\frac{d}{dt}E\left(h(X(t))\right) = \frac{d}{dt}\int_{-\infty}^{\infty} h(x)P(x,t)dx = \int_{-\infty}^{\infty} h(x)\frac{\partial P(x,t)}{\partial t}dx$$

$$E\left(\frac{d}{dt}h(X(t))\right) = E\left(\left(\frac{dh}{dX}dX + \frac{1}{2}\left(\frac{d^2h}{dX^2}\right)(dX)^2\right)/dt\right)$$

$(dX)^2$ *has an item of* dt *'s order*

In the case where there is no uncertainty, $(dX)^2$ **becomes order of** $(dt)^2$ **and** $\frac{1}{2}\left(\frac{d^2h}{dX^2}\right)(dX)^2$ **goes to 0, it becomes a general chain law**

Background

Runoff analysis introducing stochastic process theory

From Ito's stochastic differential equation to Fokker-Planck equation

$$E\left(\frac{d}{dt}h\big(X(t)\big)\right) = E\left(\left(\frac{dh}{dX}dX + \frac{1}{2}\left(\frac{d^2h}{dX^2}\right)(dX)^2\right)/dt\right)$$

property of Winnier process

$$= E\left(\frac{dh}{dX}\left(y(X,t)dt + \sigma(X,t)dw\right)/dt\right)$$

$E(dw) = 0$

$$+E\left(\frac{1}{2}\left(\frac{d^2h}{dX^2}\right)\left(y(X(t),t)^2(dt)^2 + 2y(X(t),t)\sigma(X(t),t)dtdw + \sigma(X(t),t)^2dt\right)/dt\right)$$

Ignore $(dt)^2$ order or more

$$= E\left(\frac{dh}{dX}y(X,t)\right) + E\left(\frac{1}{2}\left(\frac{d^2h}{dX^2}\right)\sigma(X(t),t)^2\right)$$

$$= \int_{-\infty}^{\infty} h'(x)y(x,t)P(x,t)dx + \frac{1}{2}\int_{-\infty}^{\infty} h''(x)\sigma(x,t)^2P(x,t)dx$$

Background

Runoff analysis introducing stochastic process theory

From Ito’s stochastic differential equation to Fokker-Planck equation

$$\frac{d}{dt}E\left(h\left(X(t)\right)\right)=E\left(\frac{d}{dt}h\left(X(t)\right)\right)$$

$$\int_{-\infty}^{\infty}\boxed{h(x)}\frac{\partial P(x,t)}{\partial t}dx=\int_{-\infty}^{\infty}\boxed{h'(x)}y(x,t)P(x,t)dx+\frac{1}{2}\int_{-\infty}^{\infty}\boxed{h''(x)}\sigma(x,t)^2P(x,t)dx$$

↓ **Partial integral** ↓ **Partial integral X 2**

$$=-\int_{-\infty}^{\infty}h(x)\frac{\partial y(x,t)P(x,t)}{\partial x}dx+\frac{1}{2}\int_{-\infty}^{\infty}h(x)\frac{\partial^2\sigma(x,t)^2P(x,t)}{\partial x^2}dx$$

Establishment against any $h(x)$:

$$\frac{\partial P(x,t)}{\partial t}=-\frac{\partial y(x,t)P(x,t)}{\partial x}+\frac{1}{2}\frac{\partial^2\sigma(x,t)^2P(x,t)}{\partial x^2}$$

Fokker-Planck equation

How to consider the uncertainty of rainfall

Using stochastic differential equation

Langevin equation

$$\frac{dx}{dt} = y(x) + \zeta'(x,t)$$

Step1:Devide the input into a random part and an average part

$$\frac{dq}{dt} = aq^b(\bar{r}(t) - q) + aq^b r'$$

Itô stochastic differential equation

$$dx(t) = y(x(t),t)dt + z(x(t),t)dw$$

Step2:Write the equations in the Ito stochastic differential equation form

$$dq = aq^b(\bar{r}(t) - q)dt + aq^b\sigma\sqrt{T_L}dw$$

Fokker-Planck equation

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial y(x)P(x,t)}{\partial x} + \frac{1}{2}\frac{\partial^2 z^2 P(x,t)}{\partial x^2}$$

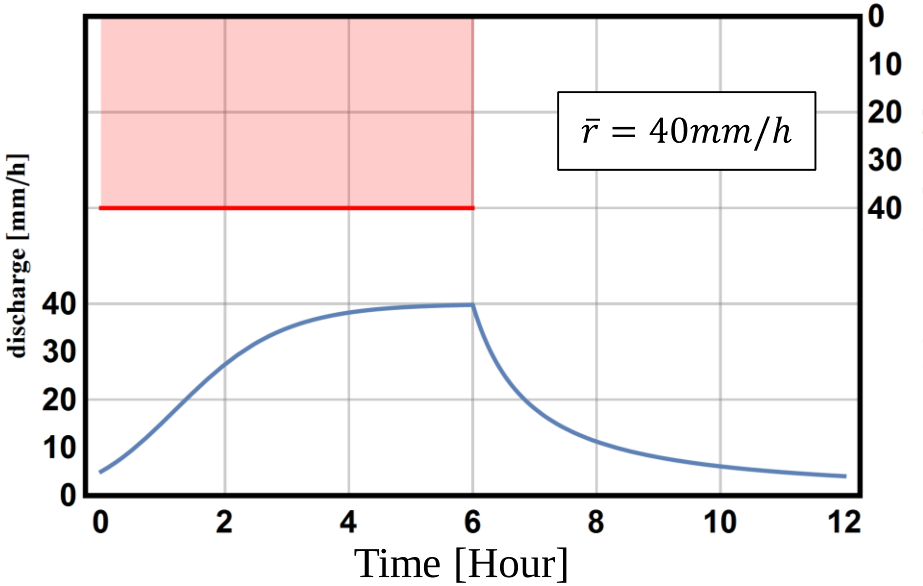
Step3:Drive the governing equations of the probability density function

$$\frac{\partial P(q)}{\partial t} + \frac{\partial aq^b(\bar{r}(t) - q)P(q)}{\partial q} = \frac{1}{2}\frac{\partial^2 (aq^b\sigma\sqrt{T_L})^2 P(q)}{\partial q^2}$$

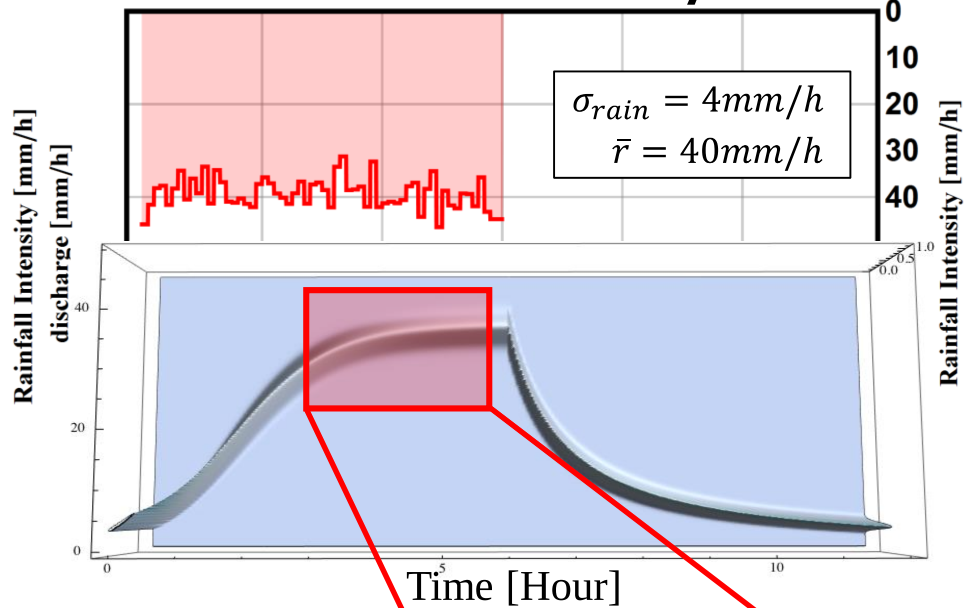
How to consider the uncertainty of rainfall

Using stochastic differential equation

Deterministic analysis



Consider the uncertainty of rainfall intensity

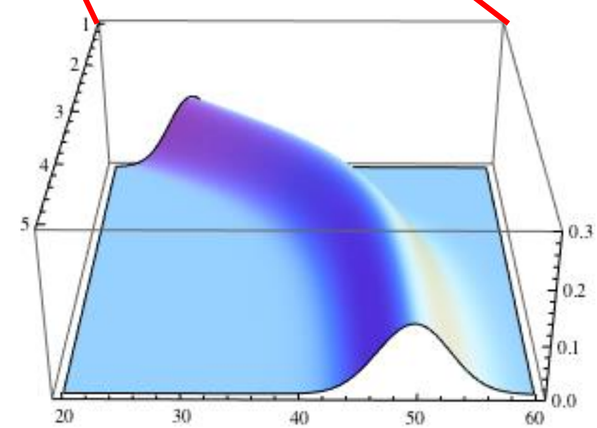


Basic equation of the deterministic model

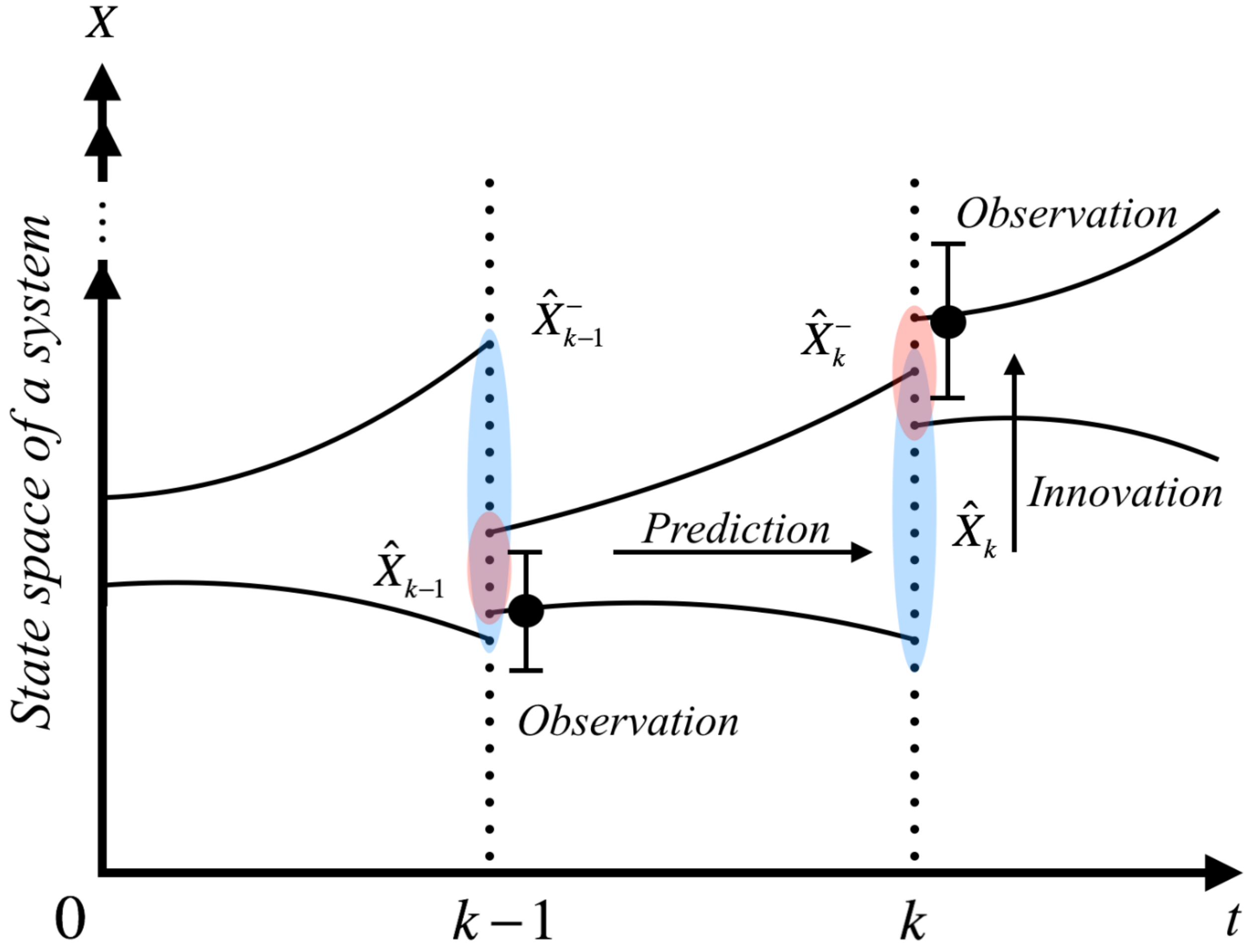
$$\frac{dq}{dt} = aq^b(\bar{r}(t) - q)$$

Fokker-Planck eq.

$$\frac{\partial P(q,t)}{\partial t} = -\frac{\partial aq^b(\bar{r}(t) - q)P(q,t)}{\partial q} + \frac{1}{2}\frac{\partial^2\left(aq^b\sigma\sqrt{T_L}\right)^2 P(q,t)}{\partial q^2}$$



The basic of filter theory (Prediction and Innovation)



Basic Concept of filtering

The basic of filter theory (Prediction and Innovation)

Step1

（1） **Update the mean value**

$$\hat{x}_k^- = A\hat{x}_{k-1}$$

（2） **Update the variance**

$$P_k^- = AP_{k-1}A^T + Q$$

Step2

（1） **Calculate the Kalman gain**

$$K_k = P_k^- H^T (HP_k^- H^T + R)^{-1}$$

（2） **Update the mean value**

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - H\hat{x}_k^-)$$

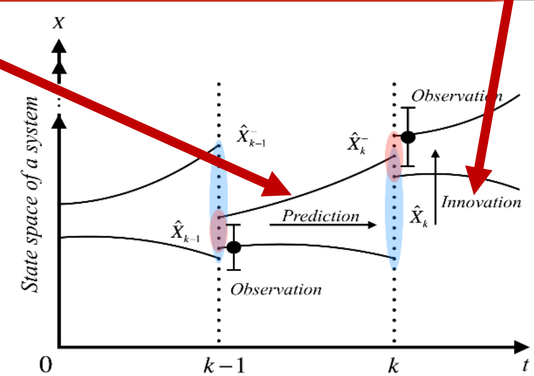
（3） **Update the variance**

$$P_k = (I - K_k H)P_k^-$$

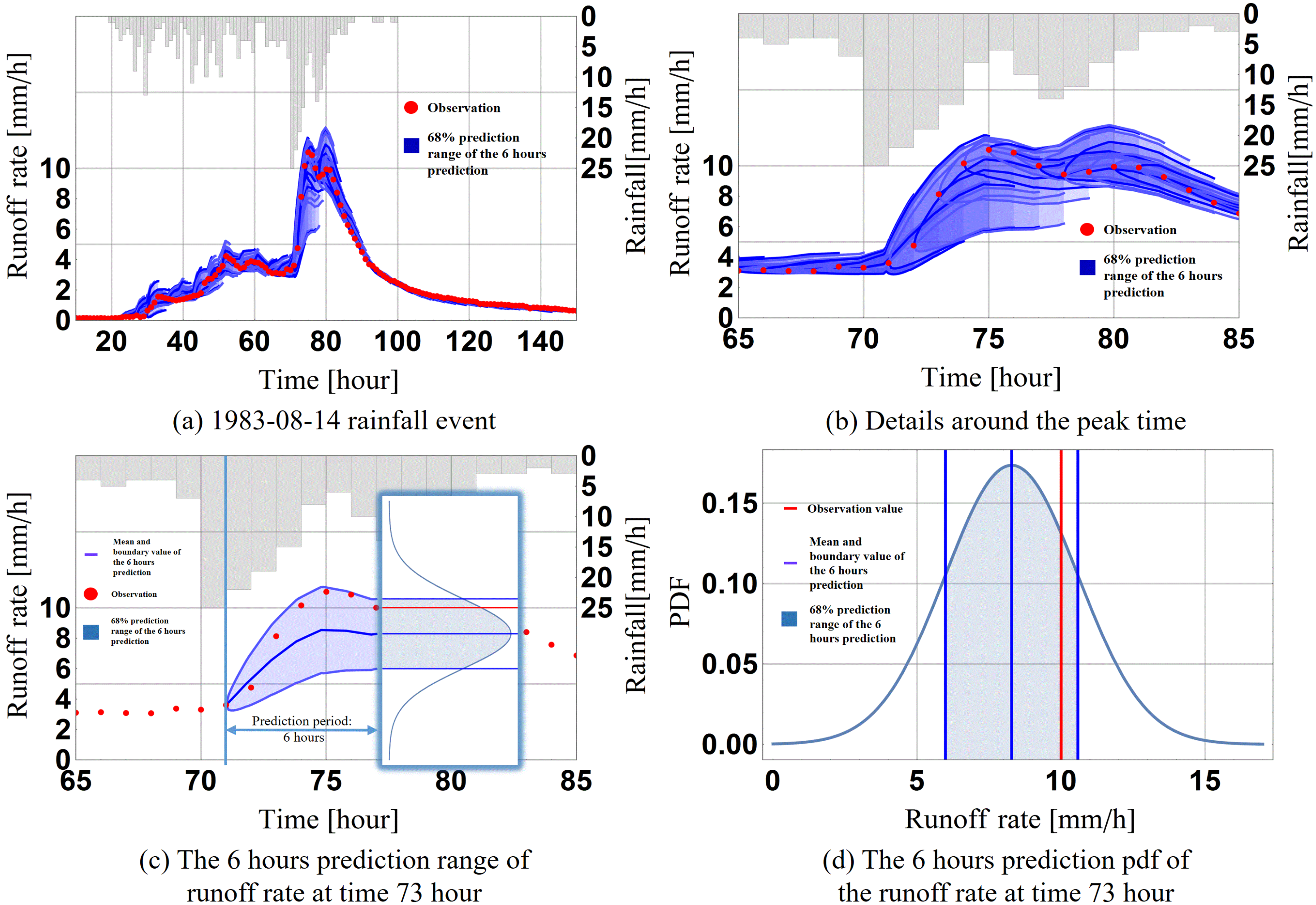
Do not have physical meanings

Input the initial condition

Algorithm of Kalman filter

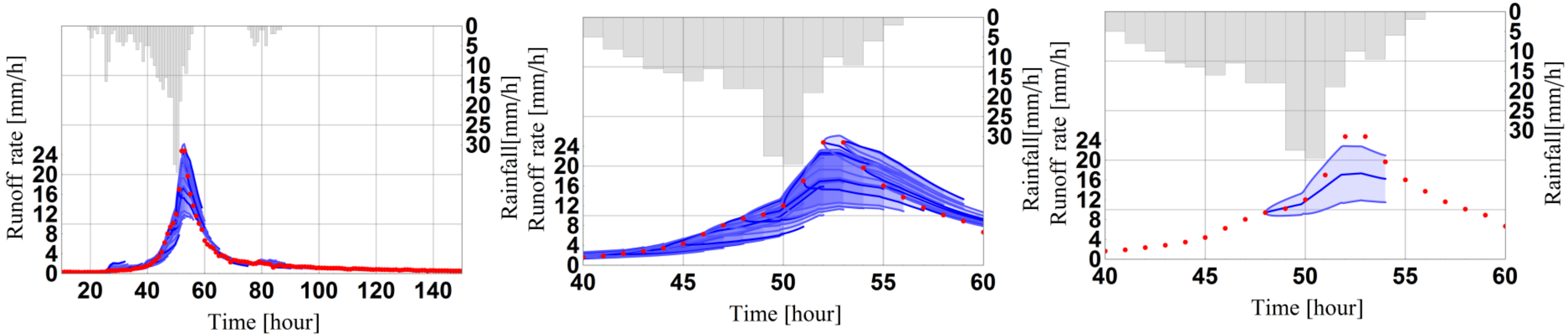


Rainfall-runoff analysis consider the uncertainty of rainfall intensity

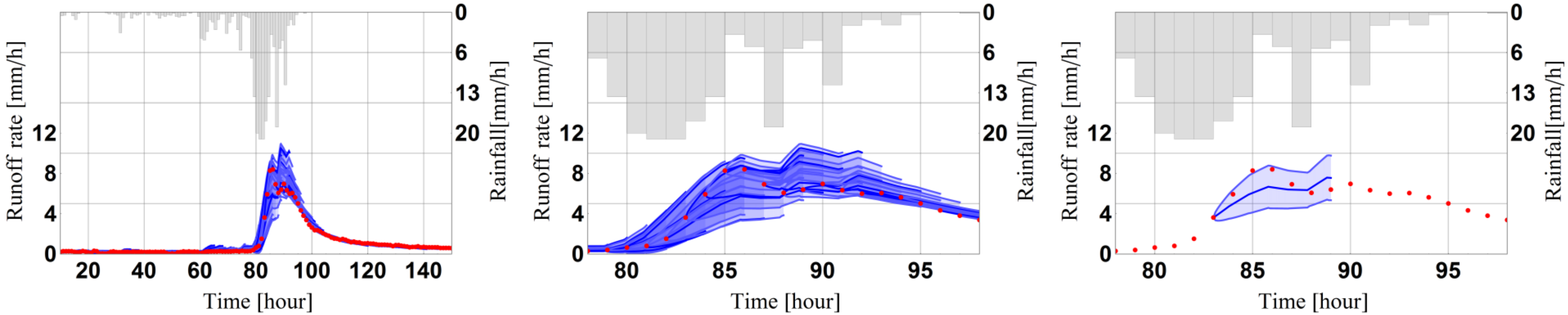


Simulation result of the 1983-08-14 rainfall event using stochastic differential equation method

Result of the new filter (1983-08-14 rainfall event)



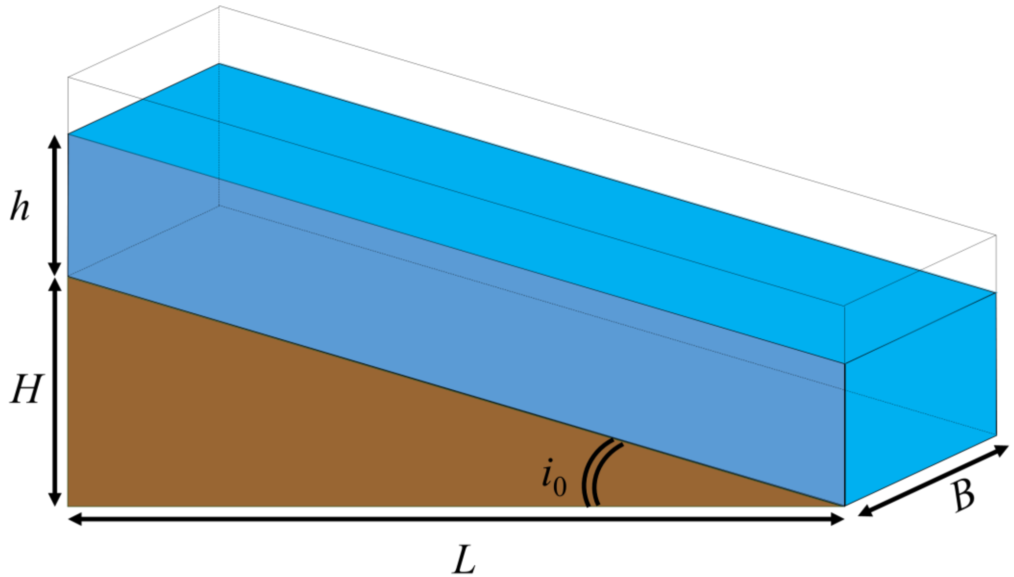
1987-07-31 rainfall event



1990-09-19 rainfall event

Some other results of the new filter

One dimensional open channel simulation



Conception grahp of one dimensional open channel

Legend

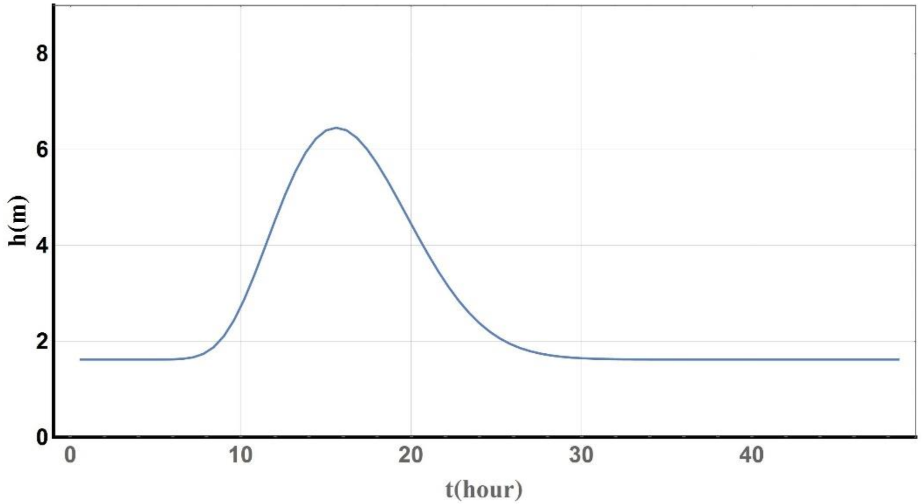
L(km)	Length of the open channel
h(m)	Water depth
q(m²/s)	Flow rate
v(m/s)	Cross-section average velocity
B(m)	Width of the open channel
i_0	Slope of the open channel
i_f	Slope of the energy loss

Governing equations

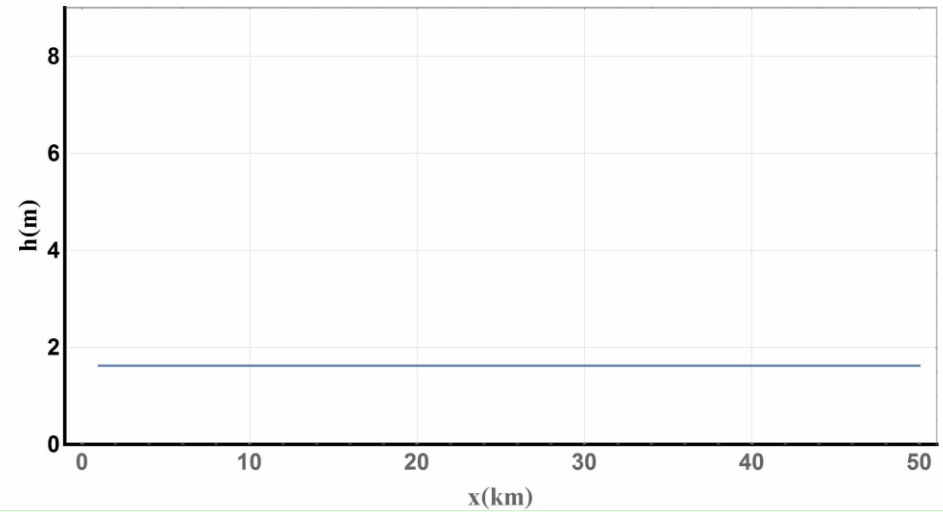
$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial (qv)}{\partial x} + gh\frac{\partial h}{\partial x} - gh(i_0 - i_f) = 0$$

One dimensional open channel simulation



Upper boundary condition



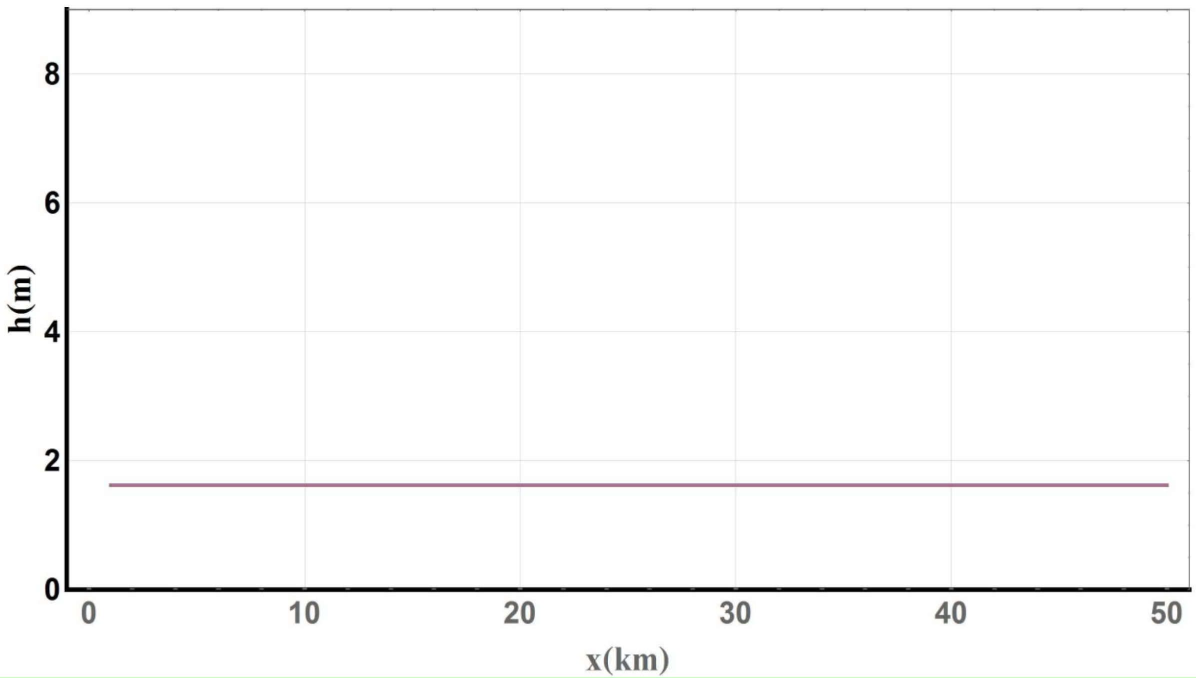
L(km)	50
T(hour)	48
Δx(km)	0.1
Δt(s)	72
B(m)	200
i_0	1/2000
i_f	Use Manning Law, rough coefficient=0.05

Left animation shows the result of a one dimensional open channel simulation. The conditions are listed above.

One dimensional open channel consider a random external force

Governing equations

$$\frac{\partial h}{\partial t}+\frac{\partial q}{\partial x}=0 \qquad \frac{\partial q}{\partial t}+\frac{\partial (qv)}{\partial x}+gh\frac{\partial h}{\partial x}-gh(i_0-i_f)=f'$$



Random simulation of one dimensional open channel under random external force

The random external force represents the uncertainty of the information of the open channel such as:
1,The uncertainty of energy loss.
2,The uncertainty of cross-section area.
3,The error caused by modelling the channel in one dimension.

The left animation showed the random simulation of the above equations.

One dimensional open channel consider a random external force

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial (qv)}{\partial x} + gh\frac{\partial h}{\partial x} - gh(i_0 - i_f) = f'$$

Solve the equations numerically

$h(x,t)$

$q(x,t)$

Same solution

$$dh(x,t) = \frac{\P h(x,t)}{\P x}dx + \frac{\P h(x,t)}{\P t}dt$$

$$dq(x,t) = \frac{\partial q(x,t)}{\partial x}dx + \frac{\partial q(x,t)}{\partial t}dt$$

Add the random external force

$$dh = g_h(h(x,t),x,t)dx + f_h(h(x,t),x,t)dt$$

$$f'dt = \sigma dw$$

$$dq = g_q(q(x,t),x,t)dx + f_q(q(x,t),x,t)dt + \sigma(q(x,t),x,t)dw(x)$$

One dimensional open channel consider a random external force

$$dh = g_h(h(x,t),x,t)dx + f_h(h(x,t),x,t)dt$$

$$dq = g_q(q(x,t),x,t)dx + f_q(q(x,t),x,t)dt + \sigma(q(x,t),x,t)dw(x)$$

Ito calculus

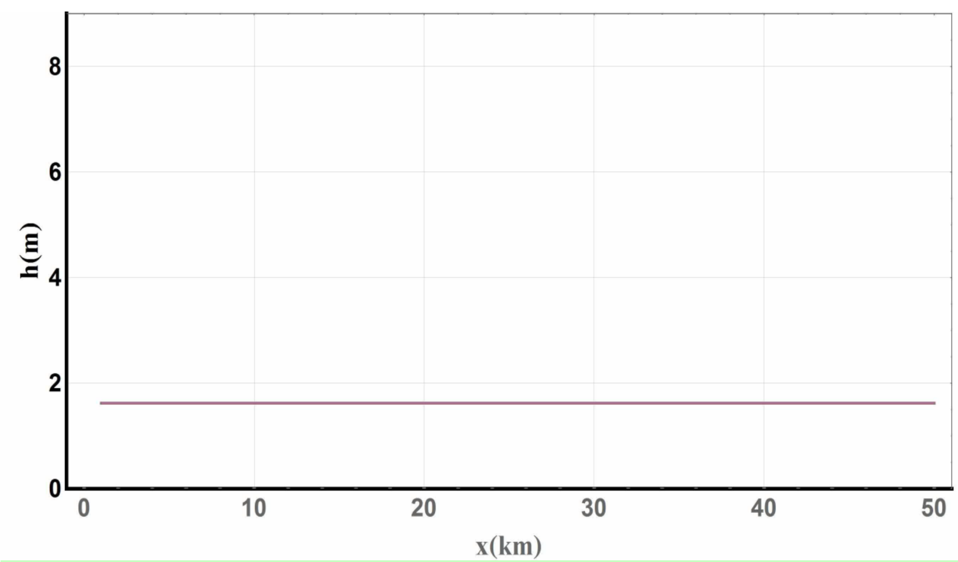
The governing equations of one dimensional open channel under random external force

$$\begin{cases} \dfrac{\partial P(h,q,x,t)}{\partial x} = -\dfrac{\partial f_h(h,x,t)P(h,q,x,t)}{\partial h} - \dfrac{\partial f_q(q,x,t)P(h,q,x,t)}{\partial q} \\ \dfrac{\partial P(h,q,x,t)}{\partial t} = -\dfrac{\partial g_h(h,x,t)P(h,q,x,t)}{\partial h} - \dfrac{\partial g_q(q,x,t)P(h,q,x,t)}{\partial q} + \dfrac{1}{2}\dfrac{\partial^2 \sigma^2(q,x,t)P(h,q,x,t)}{\partial q^2} \end{cases}$$

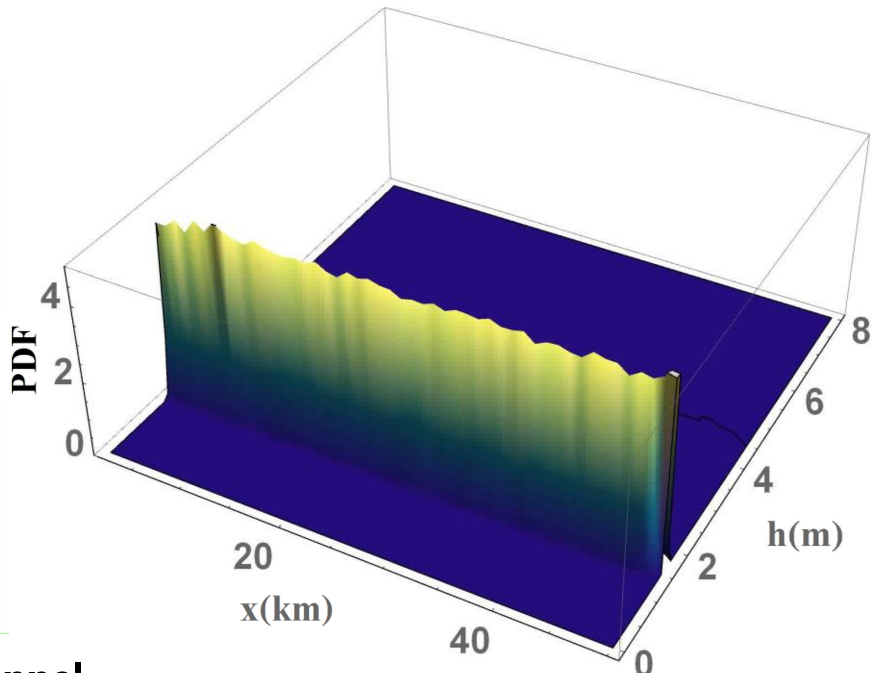
The solution of the suggested equation

$$\begin{cases} \dfrac{\partial P(h,q,x,t)}{\partial x} = -\dfrac{\partial f_h(h,x,t)P(h,q,x,t)}{\partial h} - \dfrac{\partial f_q(q,x,t)P(h,q,x,t)}{\partial q} \\ \dfrac{\partial P(h,q,x,t)}{\partial t} = -\dfrac{\partial g_h(h,x,t)P(h,q,x,t)}{\partial h} - \dfrac{\partial g_q(q,x,t)P(h,q,x,t)}{\partial q} + \dfrac{1}{2}\dfrac{\partial^2 \sigma^2(q,x,t)P(h,q,x,t)}{\partial q^2} \end{cases}$$

$\sigma = 0.01 m^2/s$

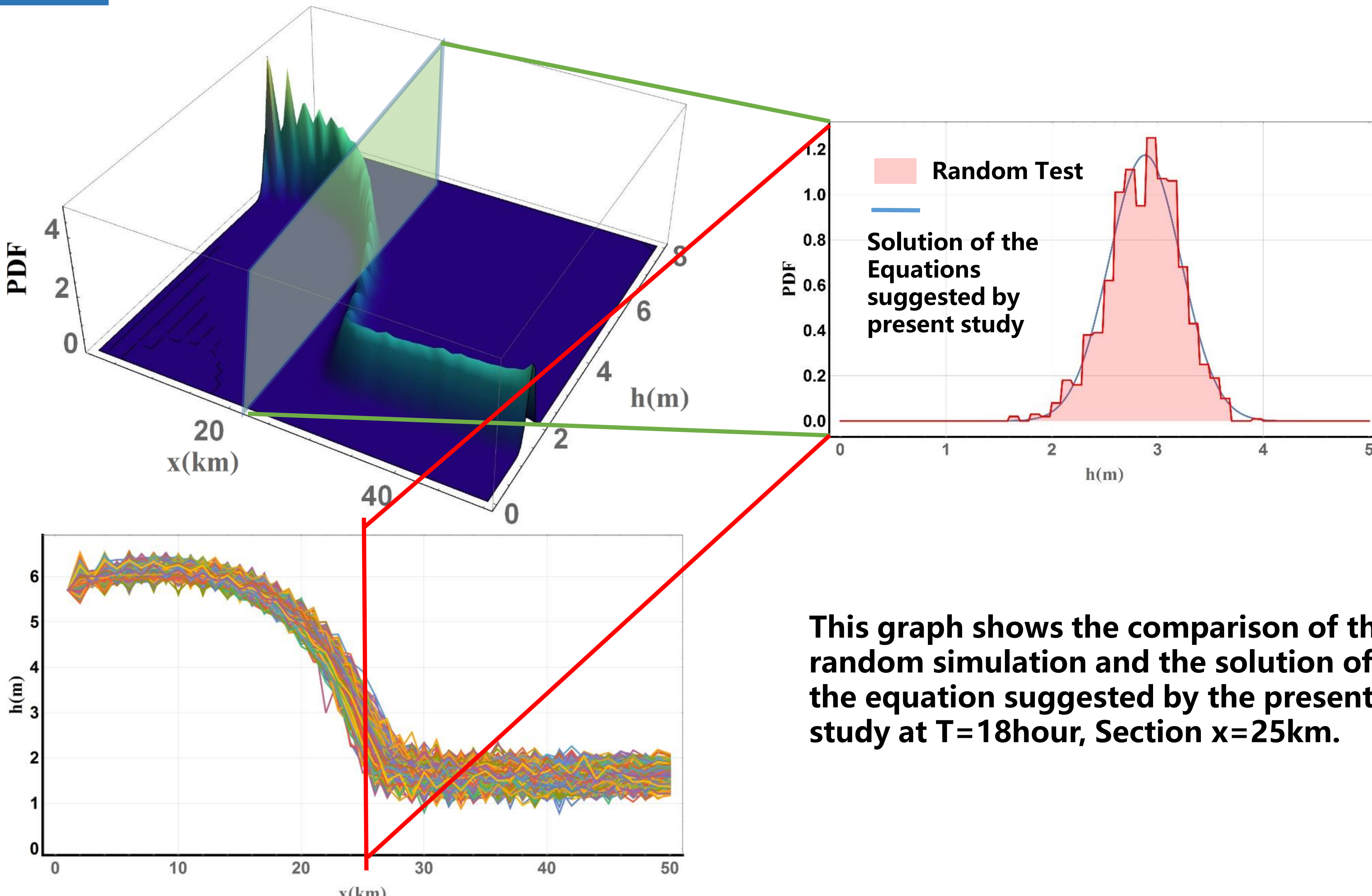


Random simulation of one dimensional open channel under random external force



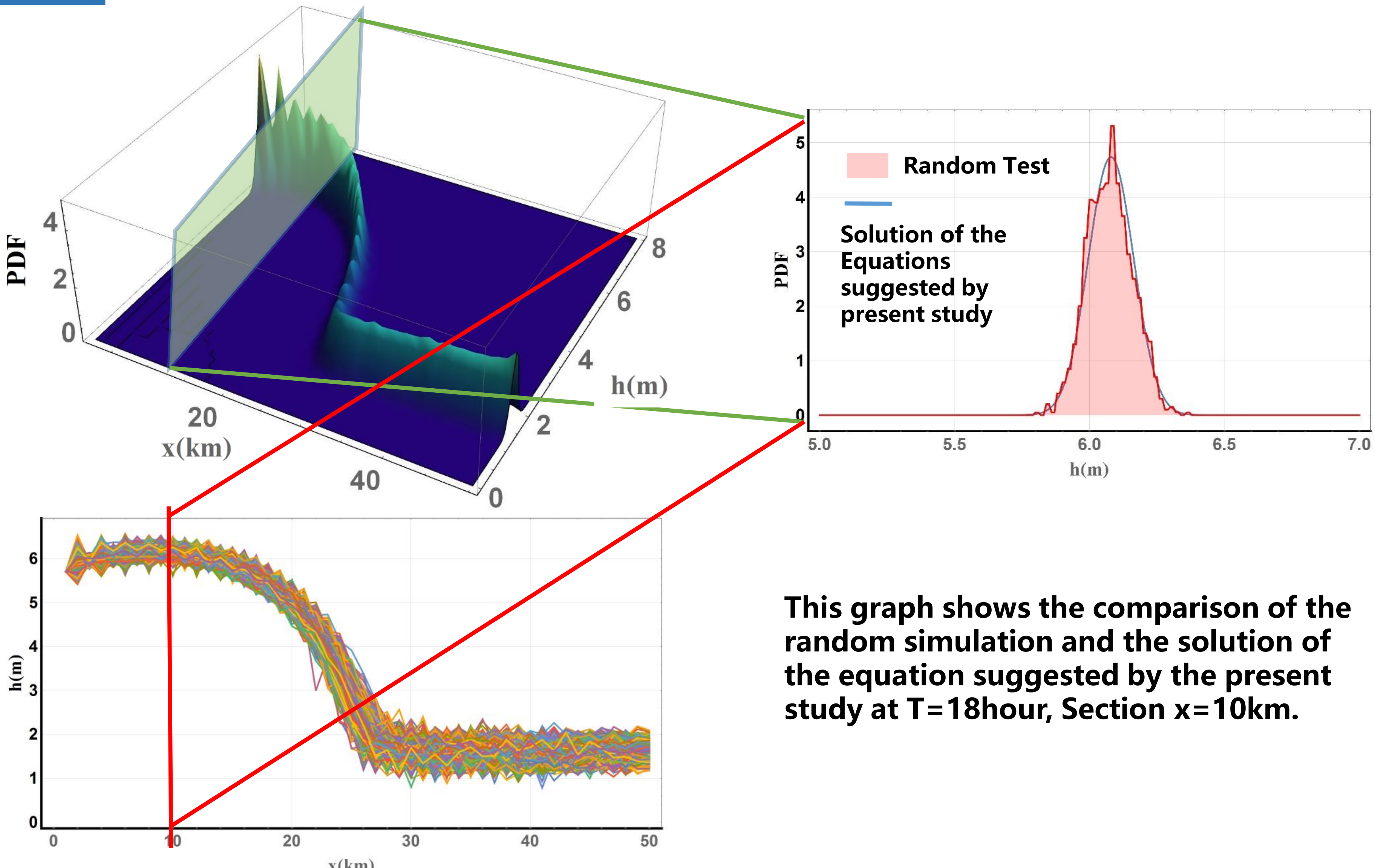
The PDF of h

The solution of the suggested equation



This graph shows the comparison of the random simulation and the solution of the equation suggested by the present study at T=18hour, Section x=25km.

The solution of the suggested equation



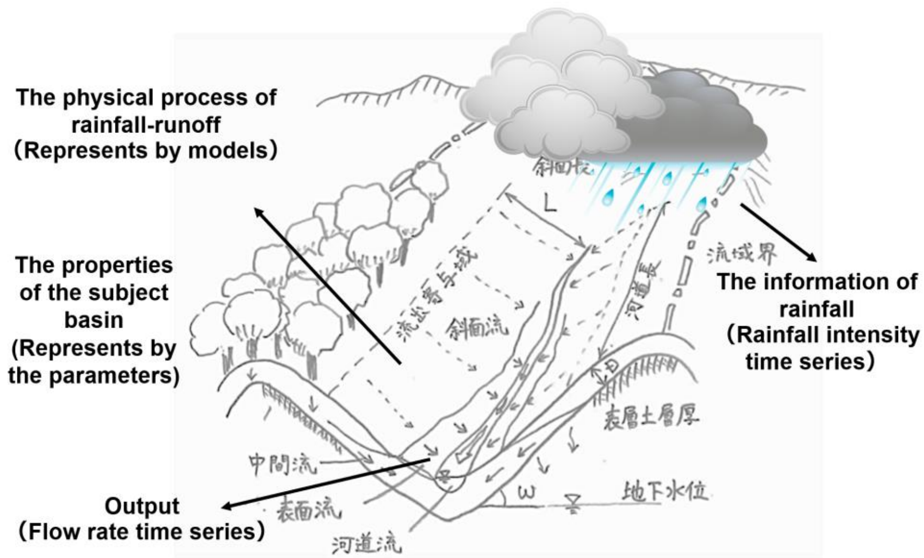
This graph shows the comparison of the random simulation and the solution of the equation suggested by the present study at T=18hour, Section x=10km.

Introduction

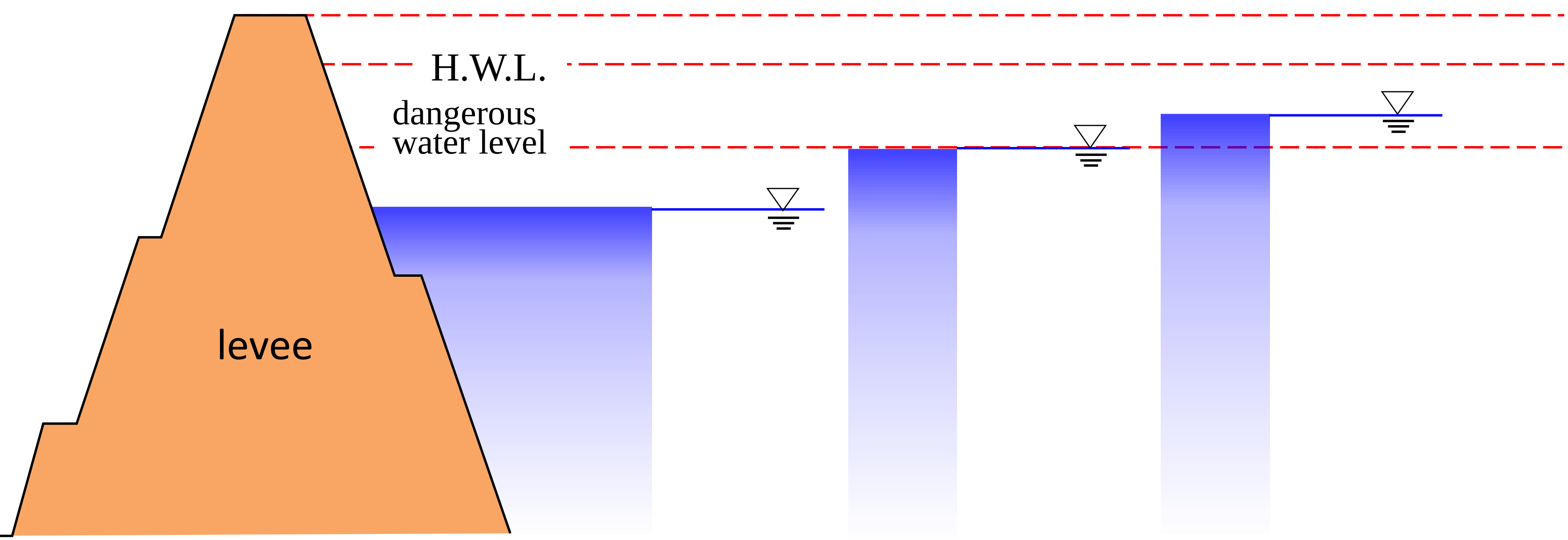
Flood forecasting

Modelized the basin, think the rainfall as input, and then we can get the time evolution of the water level.

According to the result, government can give warnings to the citizens.



Basic concept of rainfall-runoff analysis

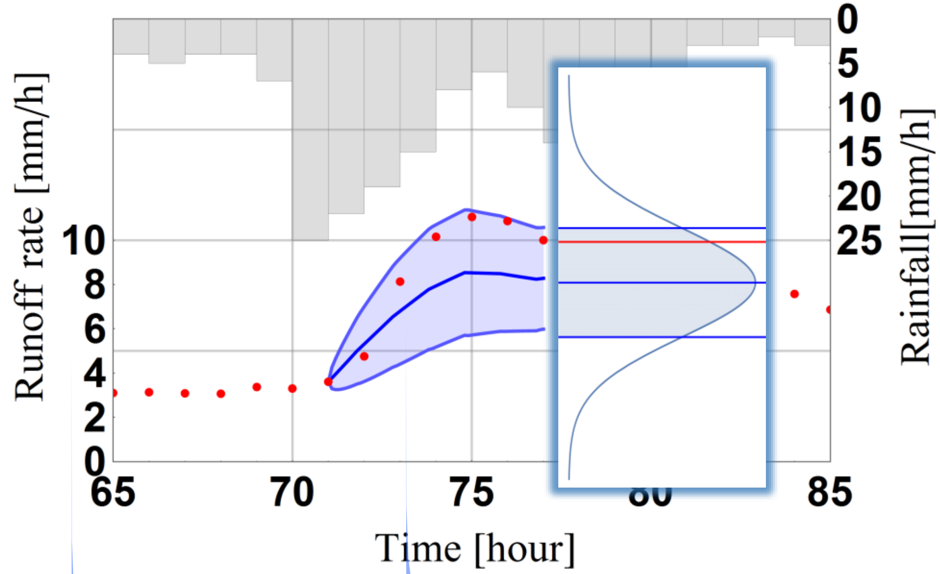


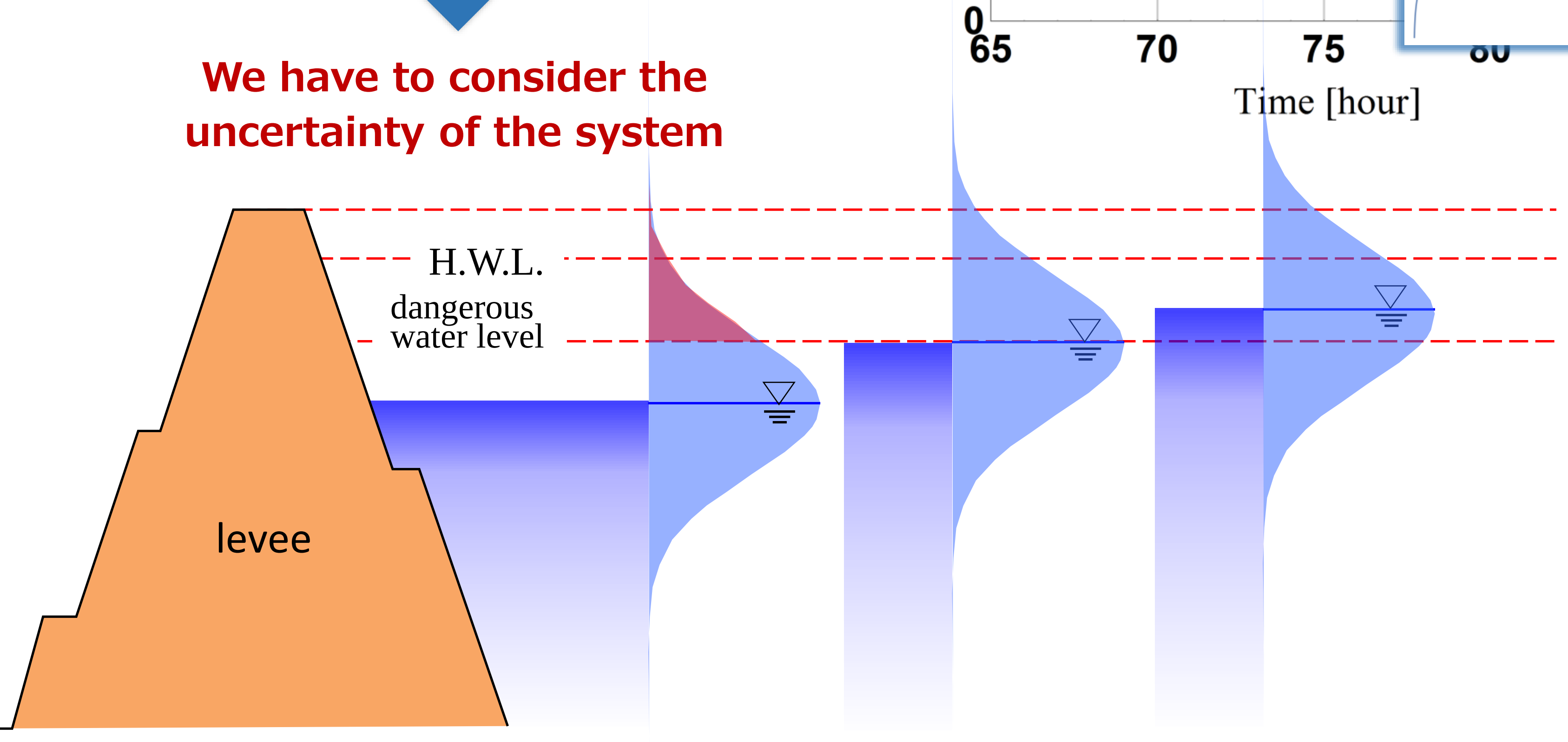
Important applications

Risk management

The most important topic of risk management is to evaluate the probability of the occurrence of disasters

We have to consider the uncertainty of the system





Chapter 4

A new theoretical method of flood forecasting and reliability evaluation of levee based on uncertainty rainfall by the stochastic process theory

- With the global climate change, the frequency of natural disaster is also change.
- **随着全球气候的**变化，自然灾**害的**发生频率也在变化
- Most of the past studies on the analysis of floods are ***determinism***. It means the analysis are only two results, stable and unstable.
- 过去关于洪水的研究分析都是基于确定论的进行的。这也就是说分析的结果只有安定和不安定两种。

2015/09 Kinugawa River (**鬼怒川破堤災害**)

国土交通省　関東地方整備局

Photo from: Ministry of Land, Infrastructure, Transport and Tourism. Kanto Regional Development Bureau.

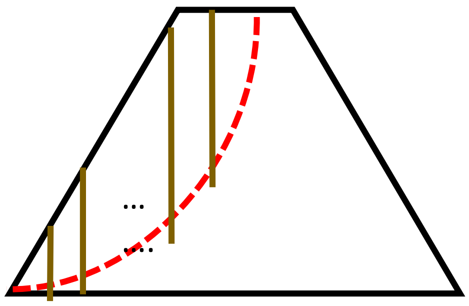
Study Results

- ❖ The stability analysis of levee with considering the uncertainty of soil parameters
- ❖ The reliability analysis of levee

❖The Stability Analysis of Levee (堤防的安定性分析)

- Circular slip method（圆弧滑动面法）

$$F_S = \frac{\sum\{c'\cdot l + (W - u\cdot b)\cos\alpha\cdot\tan\phi'\}}{\sum W\cdot\sin\alpha}$$



The cross section of levee

Fs	:	the safety factor of slope stability
c'	:	cohesion (kN/m^2 (tf/m^2))
φ'	:	friction angle of soil (^{o})
l	:	the length of the slice (m)
W	:	the weight of the slice(kN/m^2 (tf/m^2))
u	:	pore water pressure(kN/m^2 (tf/m^2))
b	:	the width of slides(m)
α	:	the inclination of the slip surface within the slice to the horizontal plane [^{o}]

- The uncertainty of soil parameters(土质参数的不确定性)
  - Because of construction method, sites, age of levee and etc.

    由于筑堤的方式，选址以及堤坝的建筑年龄。
  - However it would be not consider for the safety evaluation in generally

    但是一般来说这当进行风险评估时并不会考虑这些因素
  - The deviation of soil parameters are referred from :

    土质参数的偏差值参考：

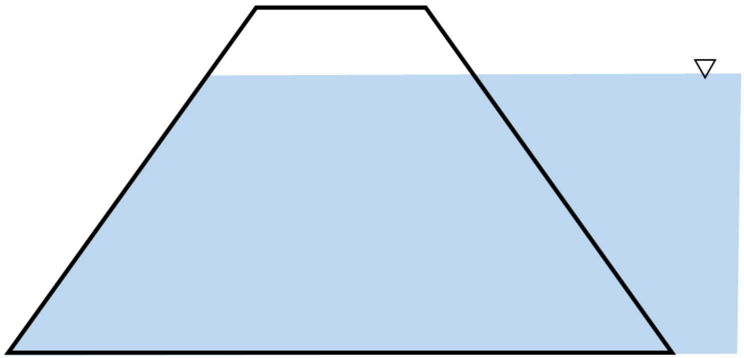
Kok-Kwang Phoon and Fred H. Kulhawy：Characterization of geotechnical variability, *Canadian Geotechnical Journal* 36(4), pp.612-624, 1999.

❖The Stability Analysis of Levee(堤防的安定性分析)

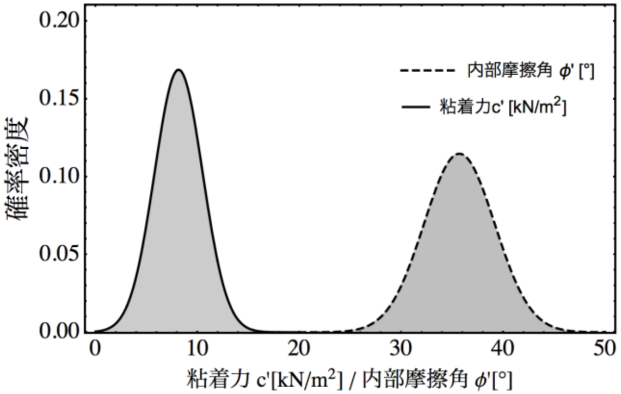
- The calculation conditions(计算条件)
  - ➢Levee(堤坝)
    - ✓Height（高程） 7.5m
    - ✓Grade（坡度） 1:2(26.4°)
- Soil parameters（土质参数）
  - ➢The unit weight of soil is $20kN/m^2$

	Cohesion（内聚力）c′	Friction angle（摩擦角）φ′
Mean value（均值）	10 kN/m^2	34 °
Coefficient of variation (%)	30	10

- The wetting plane inner levee is assumed that the same to the water level(假定堤坝内的浸润面高等于河川的水位高)



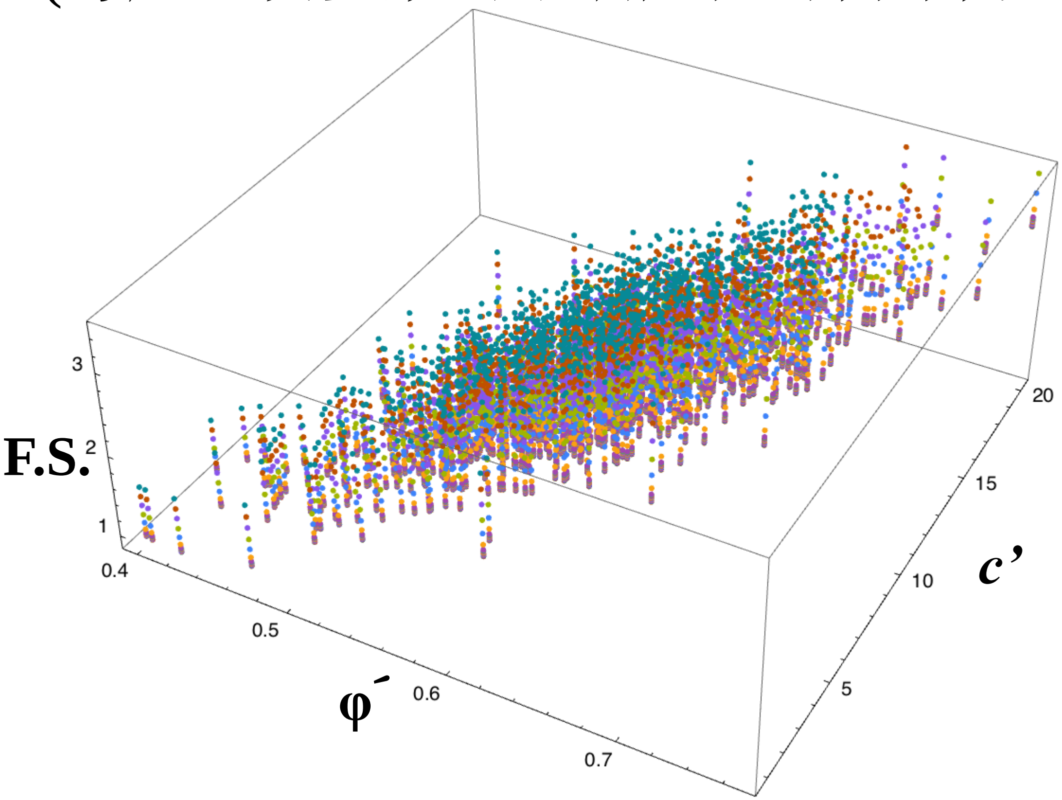
堤防横断面図

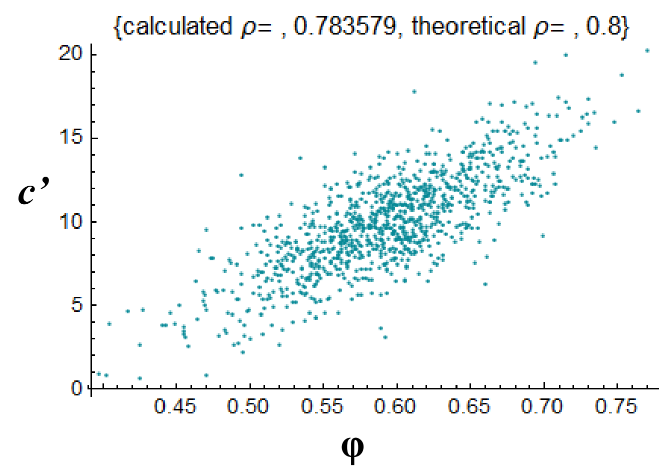


- The relationship among the cohesion, the friction angle and safety factor with considering the uncertainty of soil parameters

(考虑土质参数的不确定性时内聚力，摩擦角与安全系数的关系)

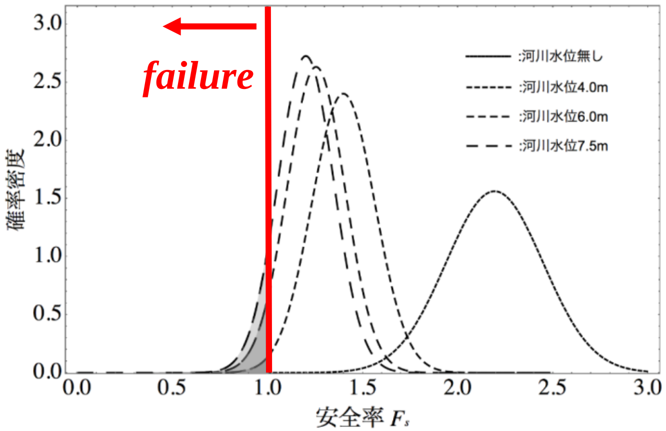
The times of calculations：10,000 times（计算次数10,000次）





The correlation of cohesion and friction angel (Correlation coefficient=0.8)

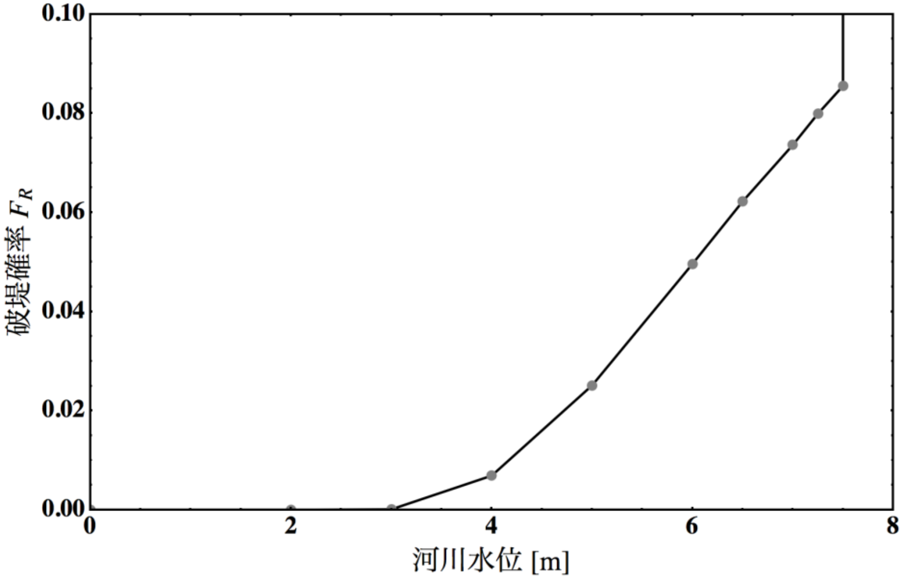
- The probability of levee broken for the certain water level(在某个确定水位决堤的概率)



The calculation method of the levee broken is as following（用以下方法计算决堤概率）

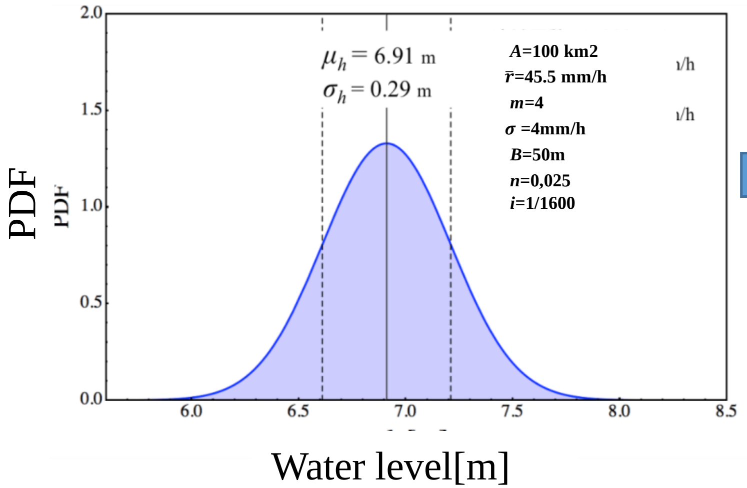
$$F_R = \frac{n}{N}$$

F_R： the probability of levee broken n：n is the case number of levee broken N：N is the number of all calculation case

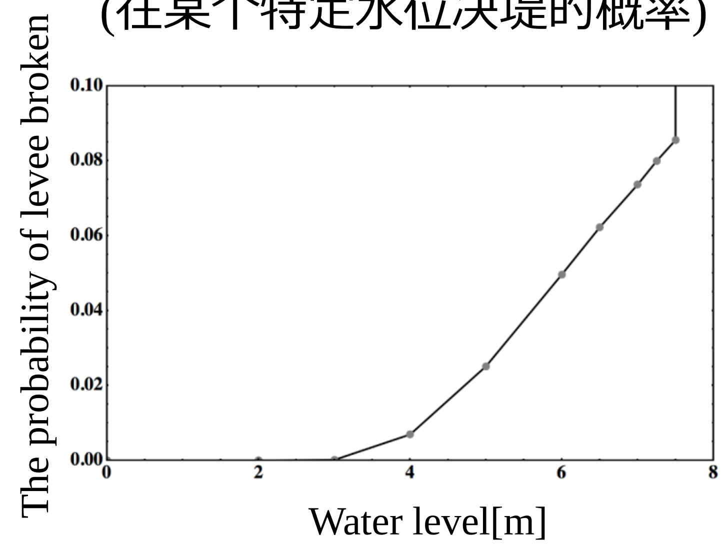


❖The Reliability Analysis of Levee (堤防的可靠性分析)

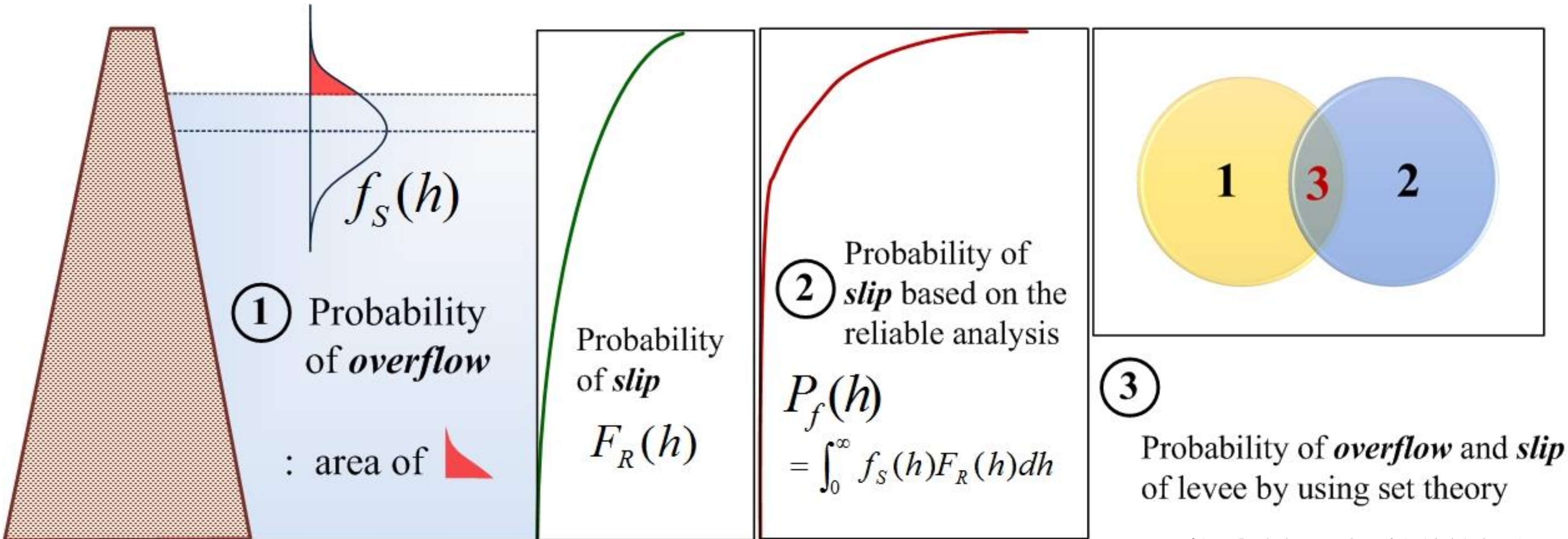
The uncertainty of water level based on the stochastic process theory
(基于随机过程理论的河川水位不确定性评价)



The probability of levee broken for a certain water level
(在某个特定水位决堤的概率)



The reliability analysis of levee
堤坝的可靠性分析



溢流概率（红色面积部分）

堤坝滑坡概率

基于可靠性分析的堤坝滑坡概率

用集合论分析的溢流和堤坝滑坡发生的概率

吉見 和紘, 山田 正， 山田 朋人：確率微分方程式の導入による降雨流出過程における降雨の不確実性の評価， 土木学会論文集B1(水工学)，59，pp.259-264，2015.

❖The Reliability Analysis of Levee

(堤防的可靠性分析)

The uncertainty of water level based on the stochastic process theory （Yoshimi et. al ,2015）

基于随机过程理论的河川水位不确定性研究（Yoshimi）

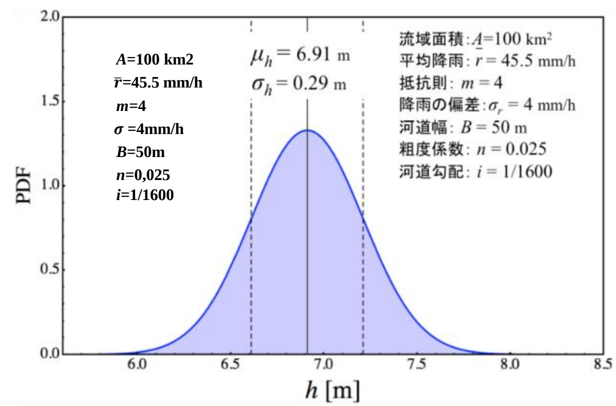
- It based on the relation between the runoff heights of stochastic differential equation and the mathematic equation of Fokker-Planck to obtain the uncertainty of rainfall and runoff.

这个研究基于一个关于降雨和径流深的随机微分方程。通过解等价与这个随机微分方程的Fokker-Planck方程来得到径流深的不确定性。

$$\frac{dq_*}{dt} = a_0 q_*^b (r(t) - q_*) \longrightarrow dq_* = a_0 q_*^b (\bar{r} - q_*)dt + a_0 q_*^b \sigma \sqrt{T_L} dw$$

Fokker-Planck

$$\frac{\partial p(q_*,t)}{\partial t} = -\frac{\partial a_0 q_*^b (\bar{r} - q_*) p(q_*,t)}{\partial q_*} + \frac{1}{2}\frac{\partial^2 (a_0 q_*^b \sigma \sqrt{T_L})^2 p(q_*,t)}{\partial q_*^2}$$



❖The Reliability Analysis of Levee

s : external force
外力载荷
f_S : PDF of external force
外力载荷的概率密度函数
r : resistance force
抵抗强度
f_R : PDF of resistance force
抵抗强度的概率密度函数

- Here according to the certain water level (like H.W.L.) the failure probability would be estimated from 0 to ∞ :

根据在每个特定水位的决堤概率，对水位从0到无穷积分，可以得到总的决堤概率

$$P[R \leq s]] = \int_0^s f_R(r)dr = F_R(s)$$

- As the range of S is $s \sim s + ds$ and because the failure probability is independent for R and S like 由于S的范围是S~S+ds，又因为R，S是独立的，所以：

$$P[R \leq s \cap s < S \leq s + ds]] = f_S(s)ds \cdot F_R(s) = f_S(s)f_R(s)ds$$

- If the external force s is form -∞ (or 0) to ∞, the failure probability of the levee may be shown 若外力载荷s是从0到无穷的，那么决堤概率可以表示为：

$$p_f = \int_0^\infty f_S(s)F_R(s)ds = \int_0^\infty f_S(s)ds \int_0^s f_R(r)dr$$

$$\longrightarrow$$

$$P_f = \int_0^\infty \int_0^s f_S(s) \cdot f_R(r)drds = \int_0^s \int_0^\infty f_S(s) \cdot f_R(r)dsdr$$

s : external force
外力载荷
f_S : PDF of external force
外力载荷的概率密度函数
r : resistance force
抵抗强度
f_R : PDF of resistance force
抵抗强度的概率密度函数

❖The Reliability Analysis of Levee

- when R is between $r \sim r + dr$, the probability $f_R(r)dr$ is the failure probability of resistance between 0 ~ ∞.

当r在r~r+dr之间，概率$f_R(r)dr$的意思是抵抗强度在0~无穷的区间里的决堤概率

$$p_f = \int_0^\infty f_S(s)F_R(s)ds = \int_0^\infty f_R(r)[1 - F_S(r)]dr$$

$f_s(s)F_R(s)$ is the mean value of failure probability when R is $r < s$
$f_R(r)[1\text{-}F_S(r)]$ is the mean value of failure probability when s is $S < r$

- The probability of levee broken from the water level 0~a certain water level

水位从0到某个特定水位的条件下的决堤概率为：

$$P_f(hS) = \int_0^\infty f_S(h_S, \sigma_S; h)\, F_R(h_R, \sigma_R; h)dh$$

fs (hs,σs ; h) : the PDF of external force h with mean h_s and standard deviation σ_S
fR (hR,σR ; h) : the PDF of resistance force h with mean h_R and standard deviation σ_R

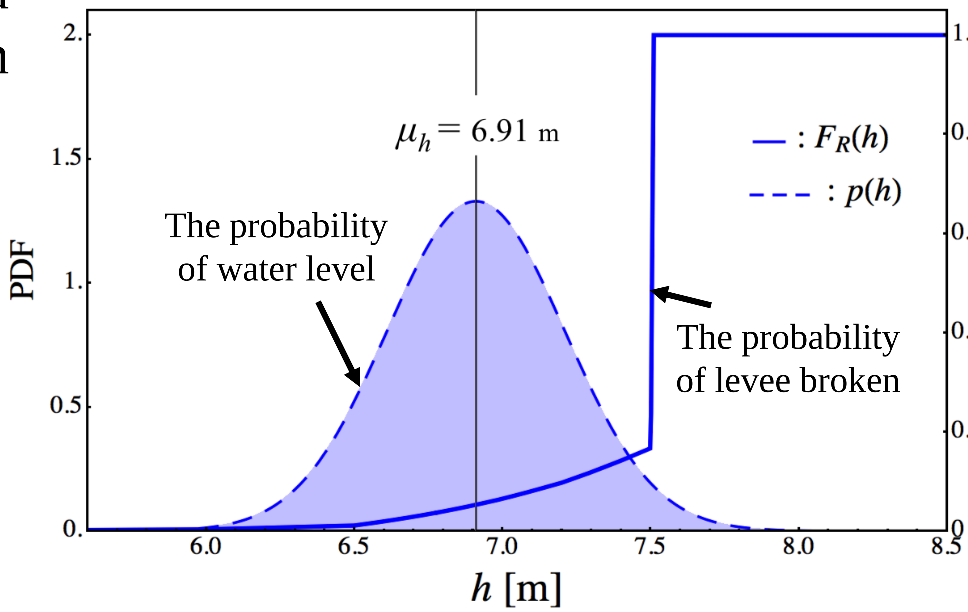
❖The Reliability Analysis of Levee

s ： external force 外力载荷	
f_S ： PDF of external force 外力载荷的概率密度函数	
r ： resistance force 抵抗强度	
f_R ： PDF of resistance force 抵抗强度的概率密度函数	

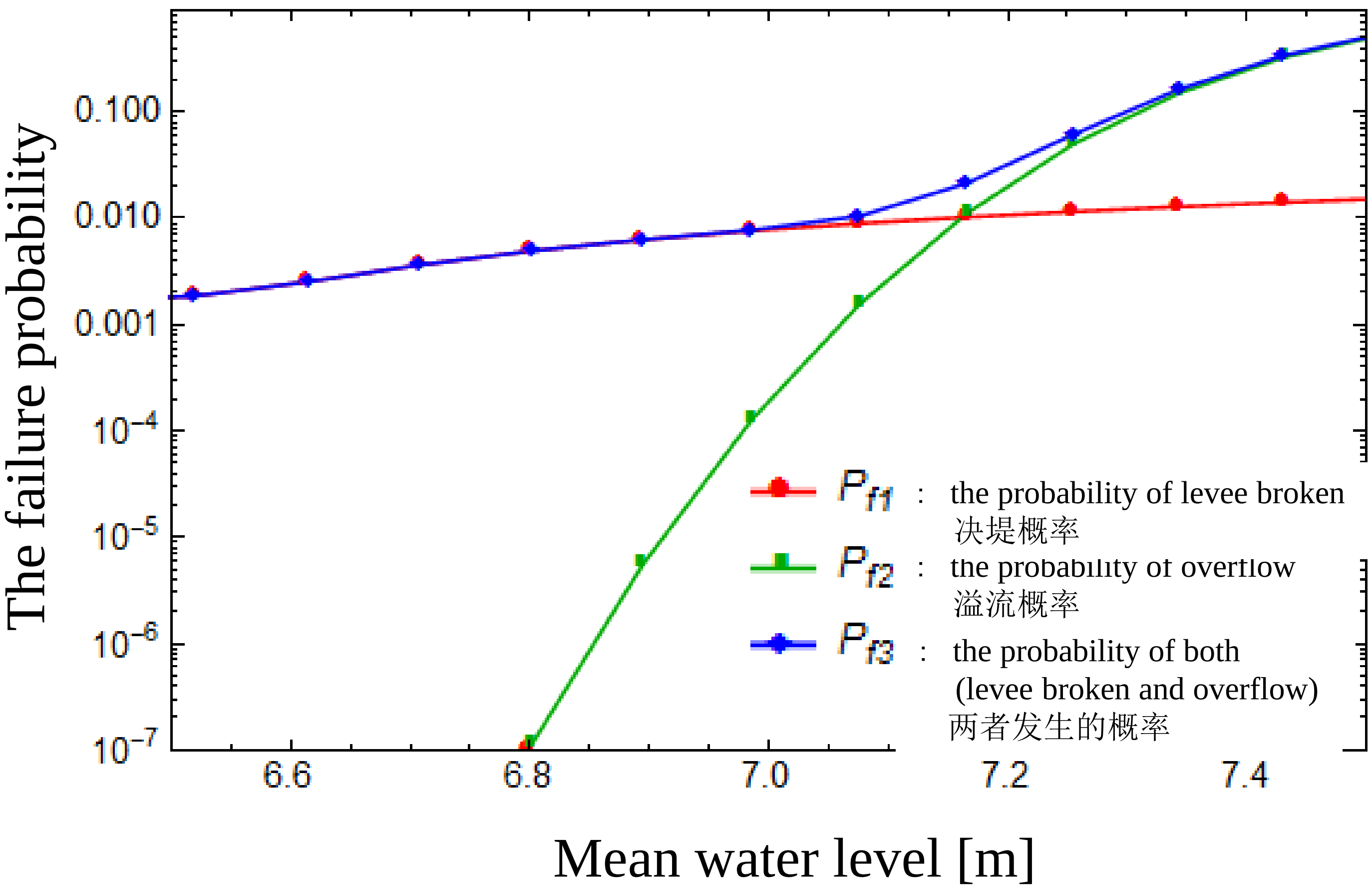
- The summation of failure probability from the water level *0* ~ H is $\overline{P_f}(H)$ and σ_S is assumed and transferred to h_S. In numerical methods

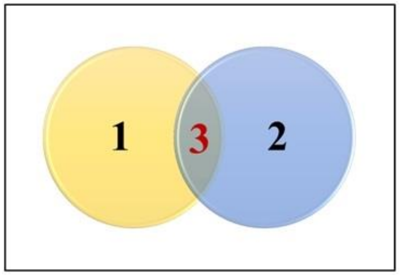
可以得出堤防在水位在*0~H*时决堤的总概率$\overline{P_f}(H)$并将之用数值方法换算成h_S。

$$\overline{P_f}(H) = \int_0^H dh_S \int_0^\infty f_S(h_S,\sigma_S;h)\, f_R(h_R,\sigma_R;h)\,dh = \int_0^\infty f_R\,(h_R,\sigma_R;h)[1-F_S(H,\sigma_S;h)]dh$$



◆ The results of the reliability analysis (可靠性分析的结果)





①the probability of levee broken
决堤概率
②the probability of overflow
溢流概率
③the probability of levee broken and overflow
两者发生的概率

堤防天端
top of levee
堤頂

Conclusions

- The safety factor is estimated then based on the uncertainty rainfall and water level, the reliability analytical solutions of the external force and the resistance force can be calculated.

可以根据降雨和水位的不确定性计算安全因子，可以对外力荷载与抵抗强度作可靠性分析，并得到解析解。

- Because of considering the inhomogeneous soil properties, the safety factor in the same conditions of water level can be different to about 2.0.

由于考虑了土壤性质的不均一性，在同一水位下安全因子的值相差可以达到2.0.

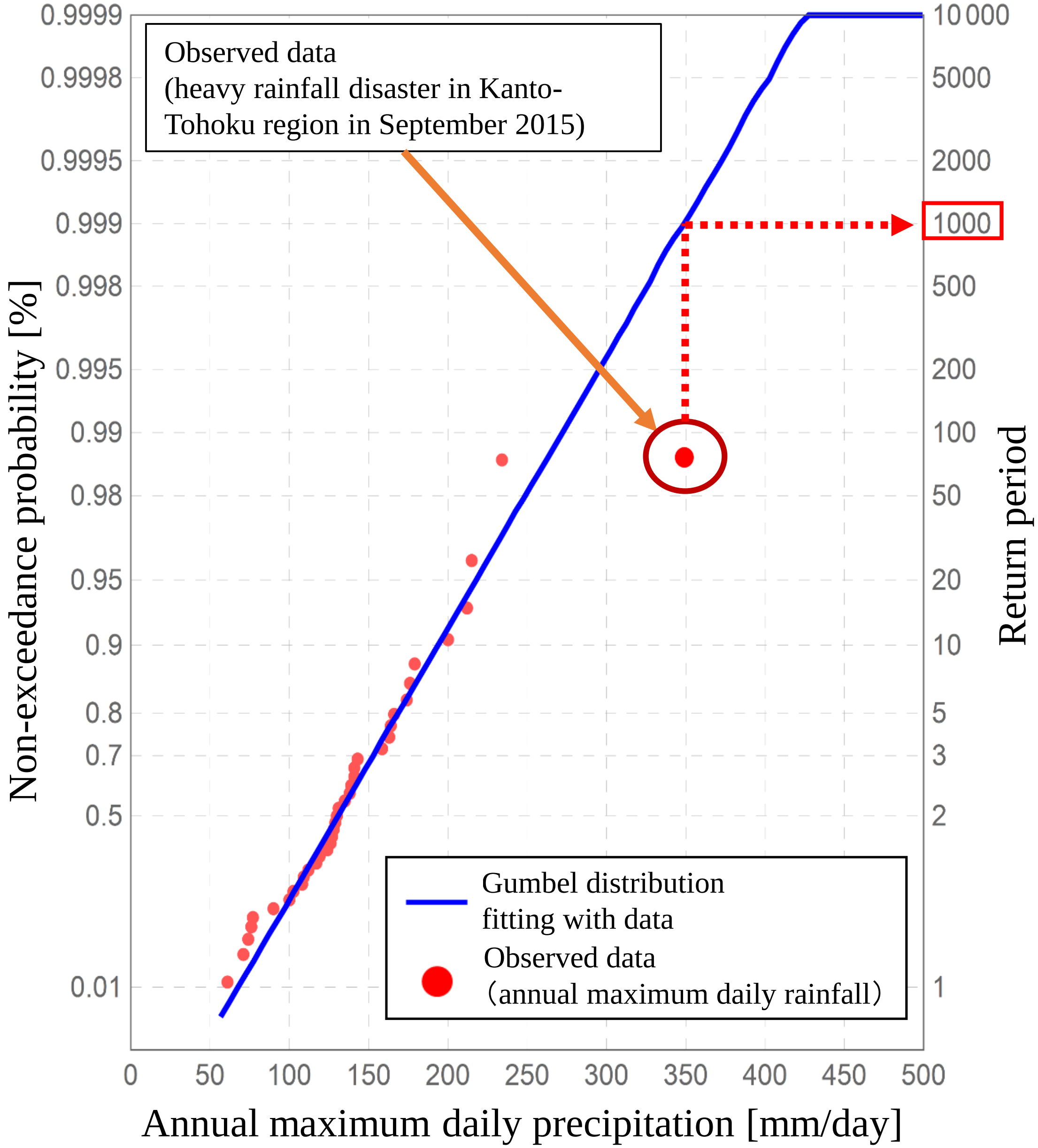
- In considering the inhomogeneous soil properties, uncertainty rainfall and water level, the reliability evolution can be known. From the 0 m to *h* of water level, the damage ratio can be estimated.

通过考虑土壤的不均一性以及降雨与水位的不确定性，可以进行可靠性评价。若对水位从0到h积分，可以得到堤防的破坏概率

Chapter 5

Uncertainty evaluation in hydrological frequency analysis introducing confidence interval and prediction interval

Difficulty of conventional hydrological frequency analysis



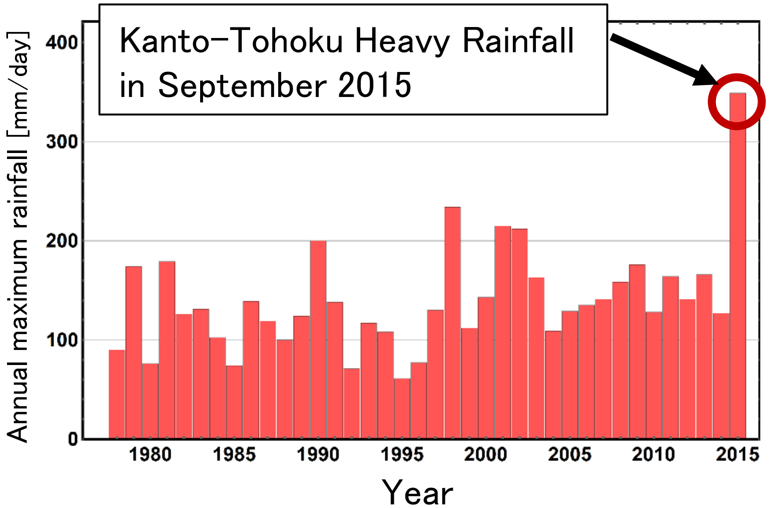


Fig. Annual maximum daily rainfall time series at Ikari observatory

Difficulty of frequency analysis caused by limited data

① Many observed data of heavy rainfall often deviate from the adopted probability distribution.

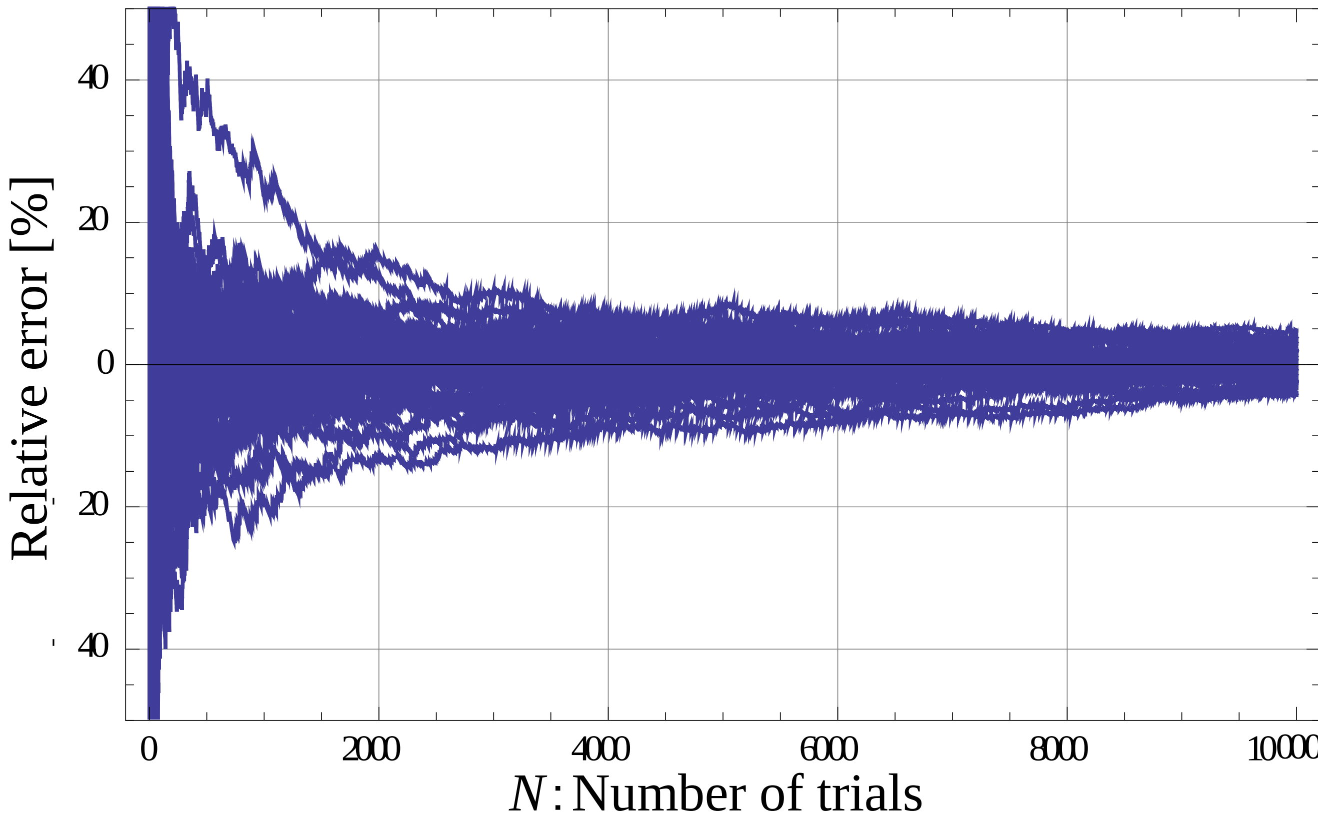
② Estimation accuracy of long-term return period decreases.

etc.

Confidence interval of extreme value statistics

Relationship between reliability of estimation and sample size

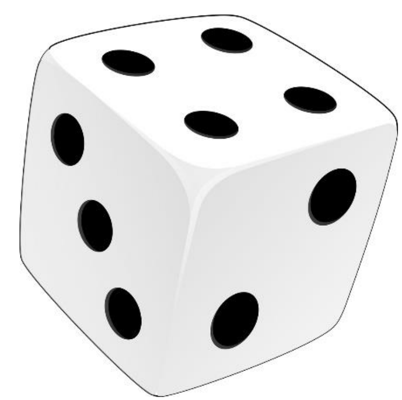
In mathematical statistics, more than several thousand data is needed to estimate parameter stably. For example, several thousand trials are needed for us to recognize probability of "1st eyes" appearing in a dice is "1/6".



Relative error[%] =

$$\frac{(M/N)-(1/6)}{(1/6)} \times 100\ [\%]$$

M : A roll of the dice,
N : Number of trials



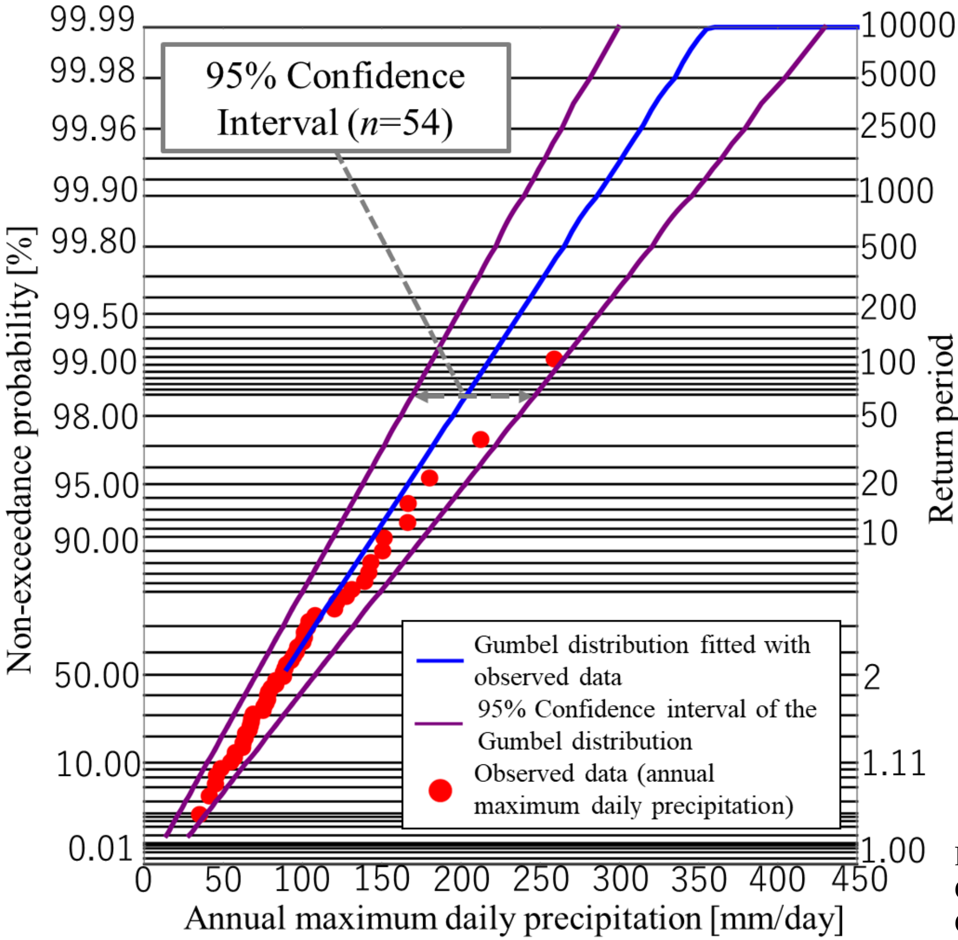
The result of this simulation suggests that extreme hydrological data for several thousand years are required to estimate the parameters of the frequency analysis model stably.

Confidence interval of extreme value statistics

An outline of the confidence interval of probability distribution model

【Definition】 The range where the probability distribution derived from N ensemble sample extracted from the same population

For example, the 95% confidence interval means that about 95% of the N probability distribution models are included. for this reason, the 2.5 percentile value of the probability hydrological distribution is on the 95% lower confidence limit line and the 97.5 percentile value is on the 95% upper confidence limit line.



Formulation of confidence interval for probability distribution model

$$P(L < Y(X) < U) \geq 1 - \beta$$

We denote the CDF fitted with the samples $\{X_1, X_2, \ldots, X_n\}$ as $Y(X)$. At this time, the interval $[L,U]$ is defined as $100(1\text{-}\beta)$% confidence interval of $Y(X)$.

U: upper confidence limit value,
L: lower confidence limit value,
β: significance level,
$1\text{-}\beta$: confidence coefficient

Formulation of **coverage probability**

$$= P(L < Y(X) < U)$$

【Definition】The rate at which probability distribution models obtained from each ensemble sample fall within the confidence interval

Fig. Observed data of annual maximum precipitation at Yattajima Observatory and Gumbel distribution fitted these observed data, 95% confidence interval of the Gumbel distribution

Confidence interval of extreme value statistics

Relationship between confidence interval and sample size

As the number of data increases, the confidence interval narrows, and the reliability of estimation improves.

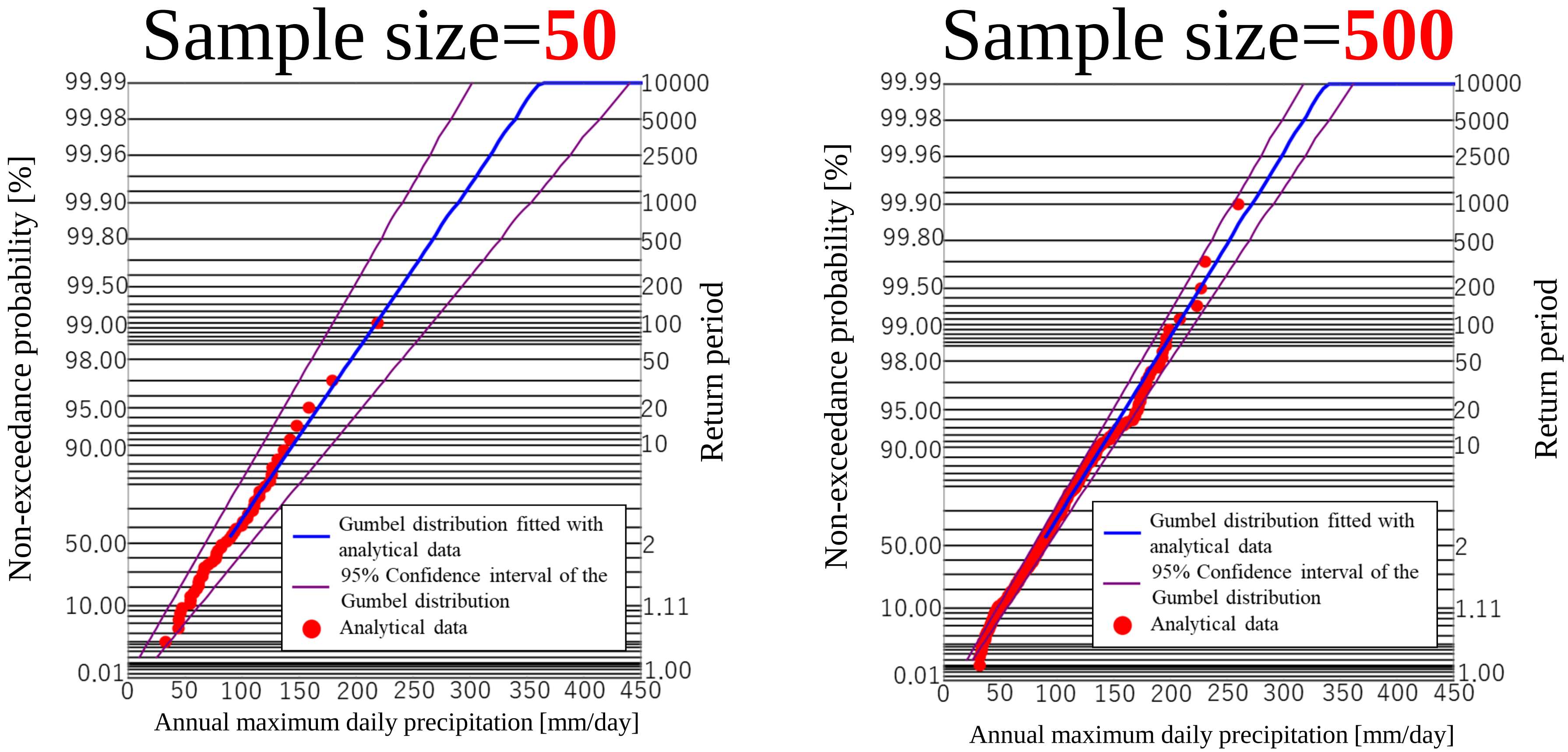


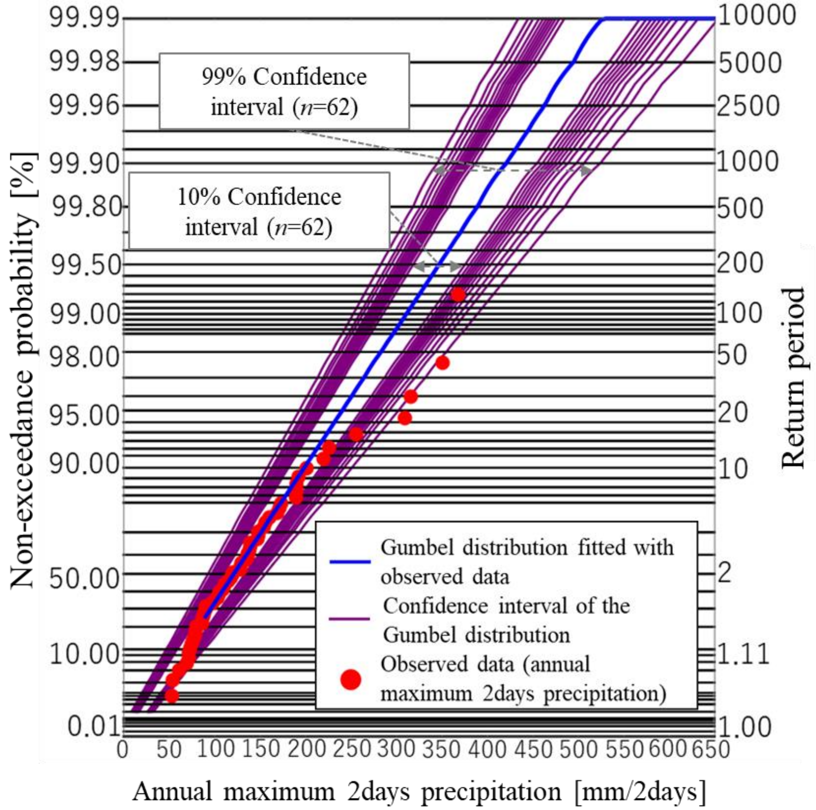
Fig. Relationship between confidence interval and sample size

Analytical data (red dots) on both probability papers are random numbers according to the Gumbel distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, 95% confidence intervals were written in both probability papers.

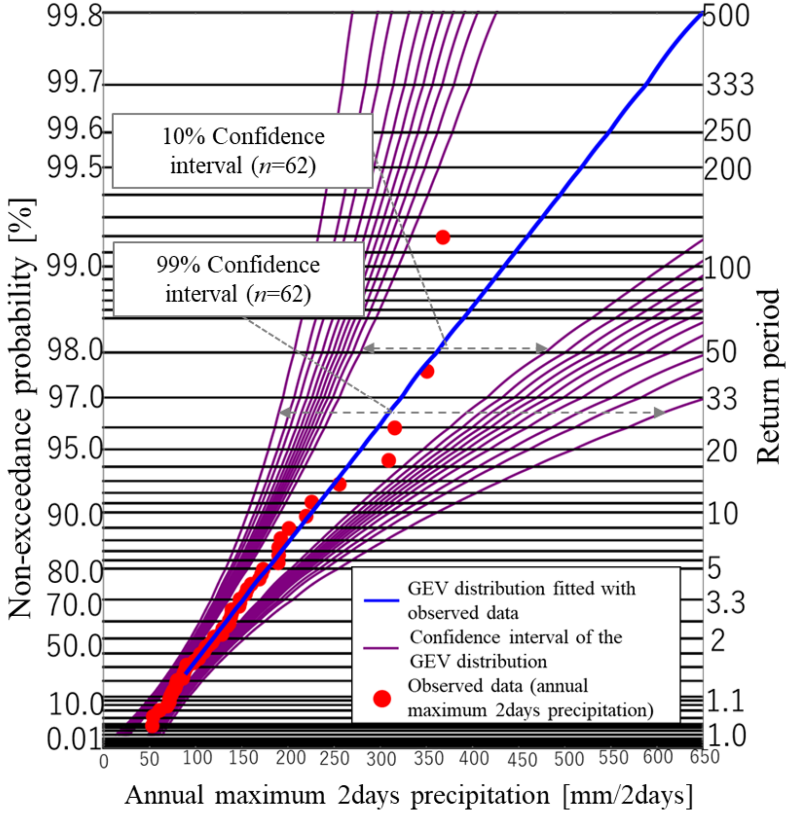
Confidence interval of extreme value statistics

Relationship between confidence interval and probability distribution models

Adoption of Gumbel distribution



Adoption of GEV distribution



Gumbel distribution (2 Parameters) : It shows good fit to the maximum value of normal year and the corresponding confidence interval is narrow.

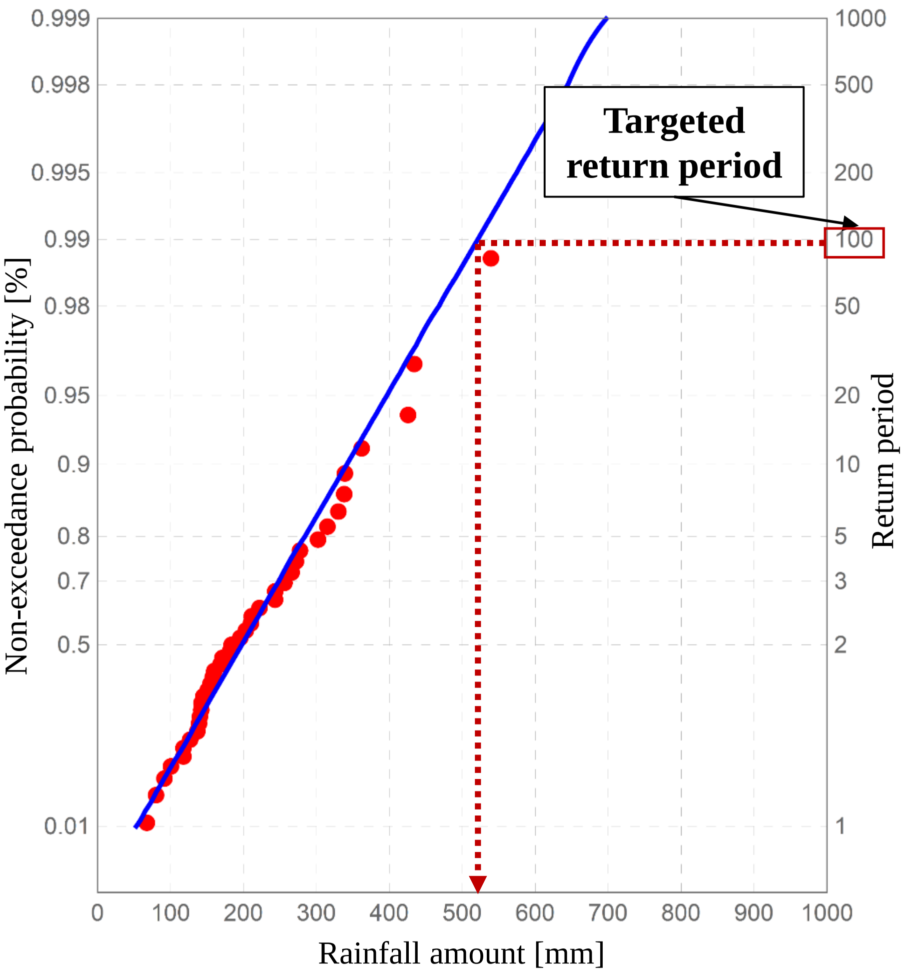
Generalized Extreme Value Distribution (3 Parameters) : It shows good fit for the whole data but the corresponding confidence interval is wide.

Fig. Observed data of annual maximum 2days precipitation at Nakanojou Observatory and Gumbel (/GEV) distribution fitted these observed data, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95,99 % confidence interval of the Gumbel (/GEV) distribution

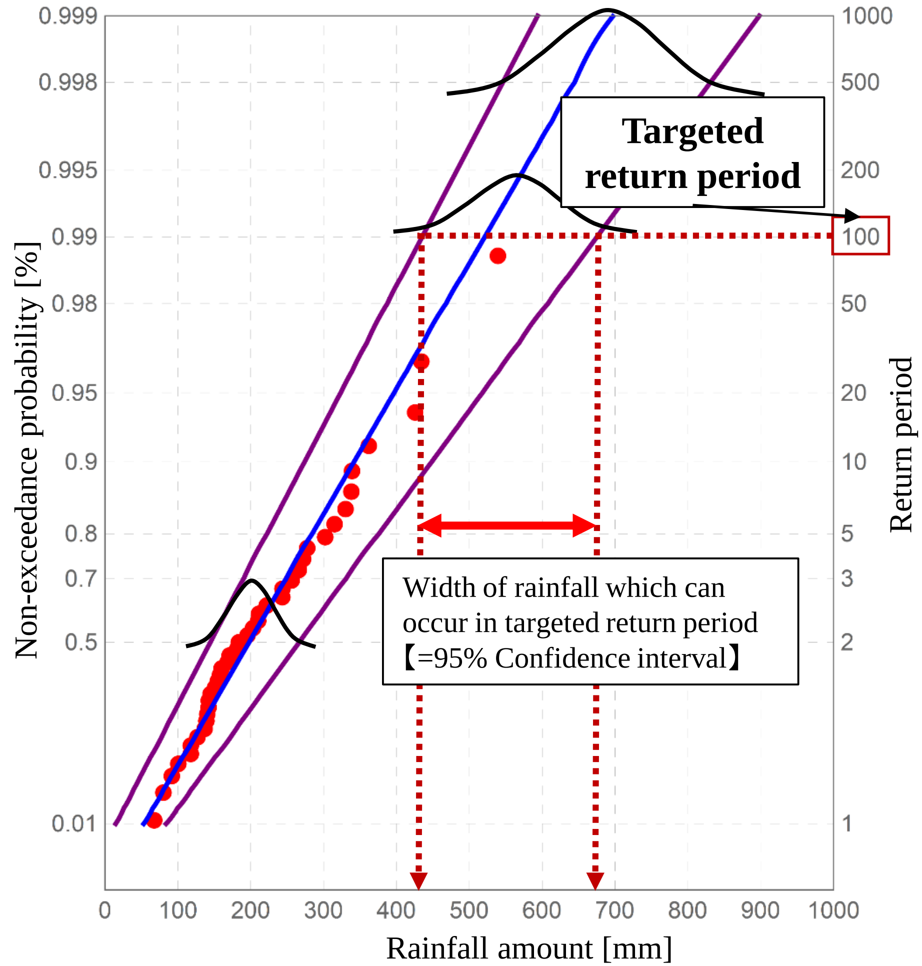
Introduction of confidence interval

By Introducing confidence interval, it is possible to intake heavy rainfall which is considered “unexpected” in flood management.

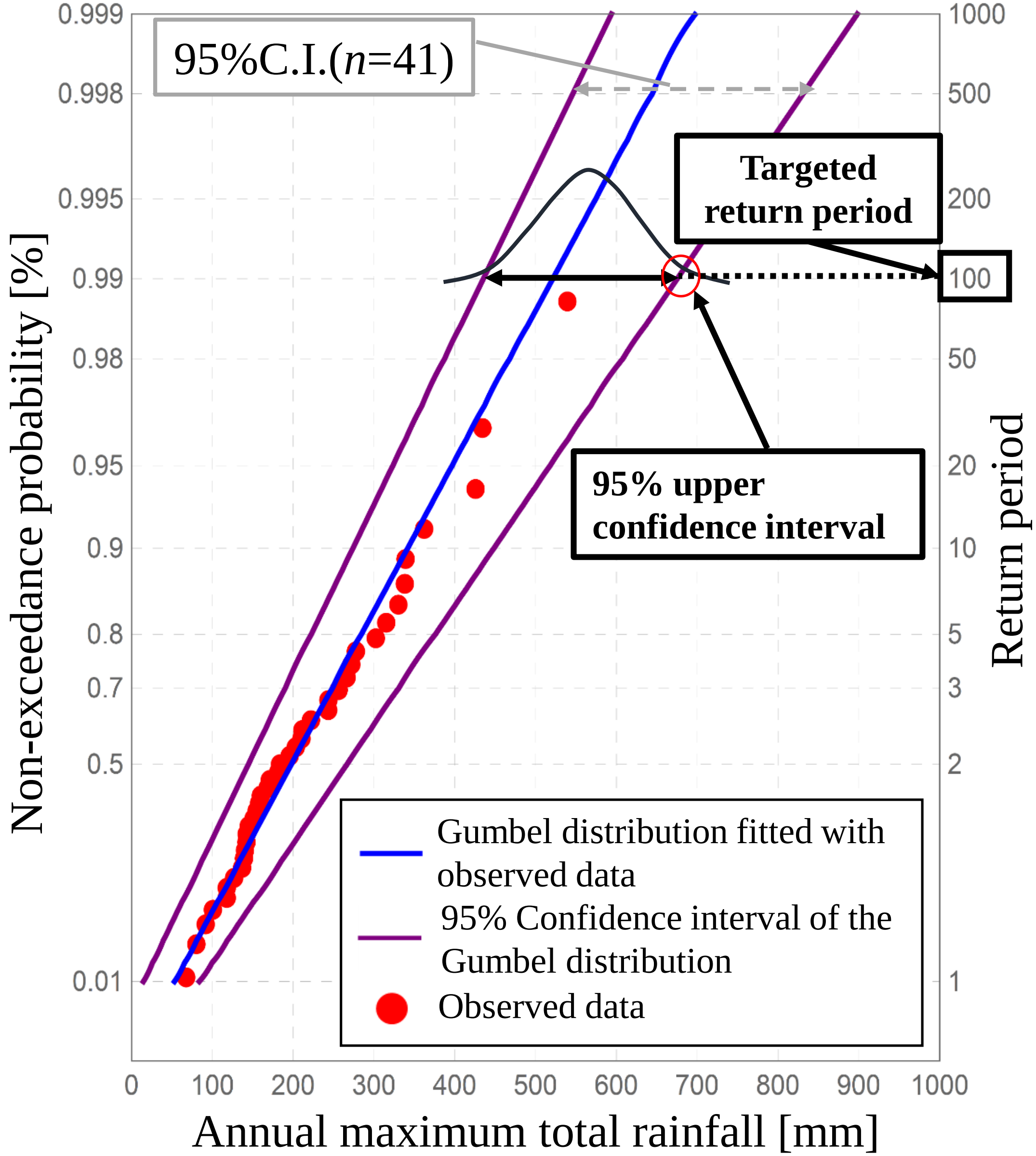
【Conventional risk evaluation】



【Risk evaluation based on C.I.】



Evaluation of heavy rainfall using confidence interval



This probability paper shows 41 observed data of annual maximum total rainfall in Kusaki Dam basin, Gumbel distribution fitting with these data and 95% confidence interval based on probability limit method test. n shows total number of observed data.

Exceedance probability of confidence limit is expressed by the product of "targeted return period" and "exceedance probability of C.I."

Exceedance probability of 95% upper confidence limit of 100-year rainfall

$$\frac{1}{100} \times 0.025$$

Return period

Exceedance prob. (95% C.I.)

$$= 2.5 \times 10^{-4}$$ (1/4000)

By considering the confidence intervals, it is possible to calculate the risk of occurrence of unprecedented heavy rain.

Relative evaluation of risk realized

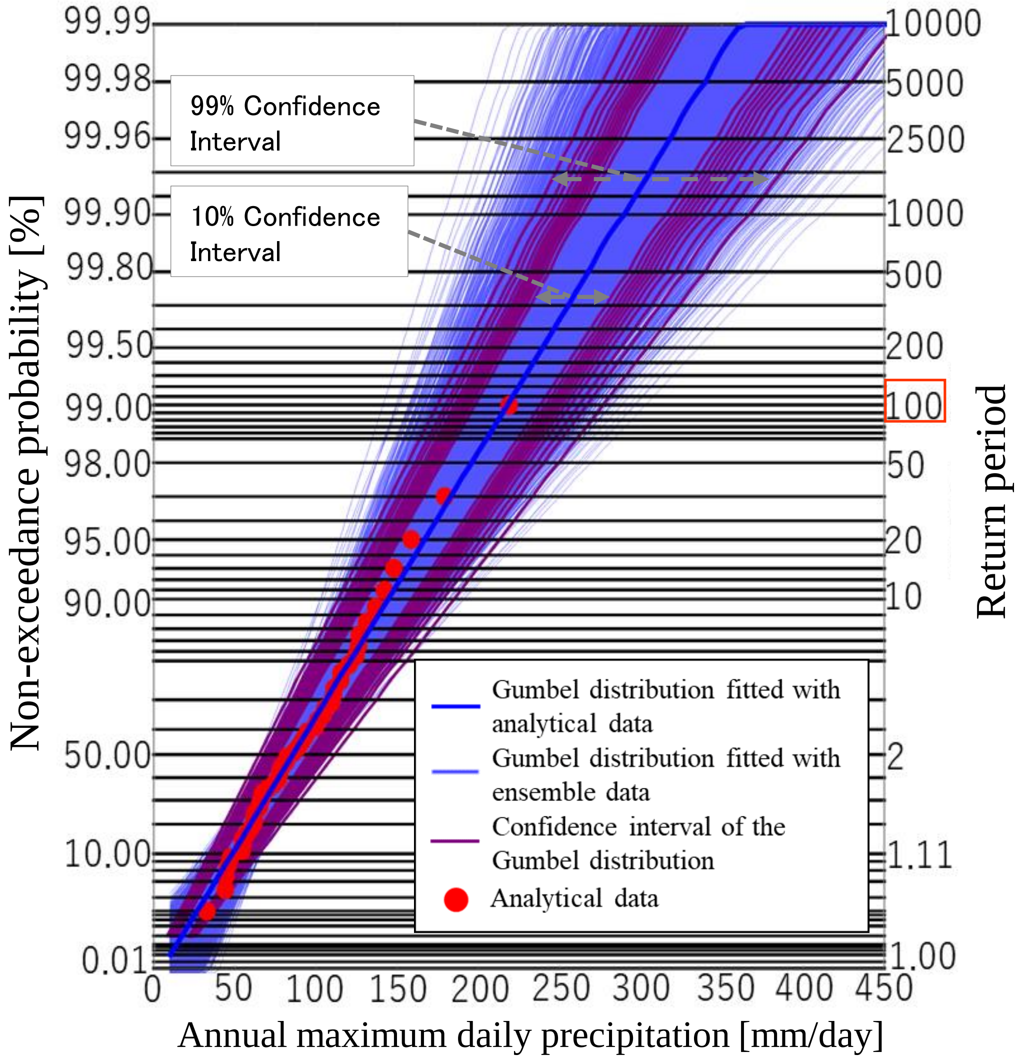
[ref : **the rate of deaths**]

traffic accident : $1/(2\times10^{4})$ [/year]

air plane accident : $1/(50\times10^{4})$) [/year]

drug accident : 1/(200万$\times10^{4}$) [/year]

Sample size n = 50
Gumbel distribution adopted



Analytical data (n=50) on above probability paper are random numbers according to the Gumbel distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, Gumbel distribution fitted with analytical data and 5000 Gumbel distribution fitted with ensemble data (n=50), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the Gumbel distribution fitted with analytical data

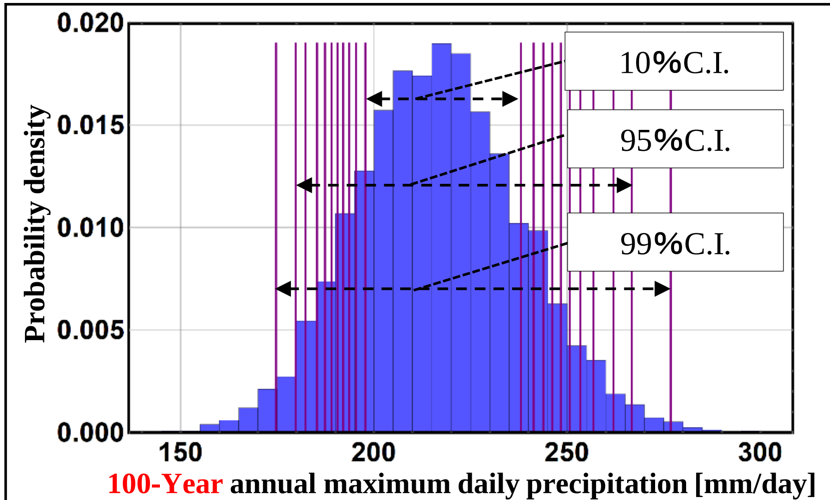


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I.[197.8, 238.0] ＝**64.7%**
Coverage probability of 95% C.I.[179.7, 266.7] ＝**95.1%**
Coverage probability of 99% C.I.[174.7, 276.8] ＝**97.4%**

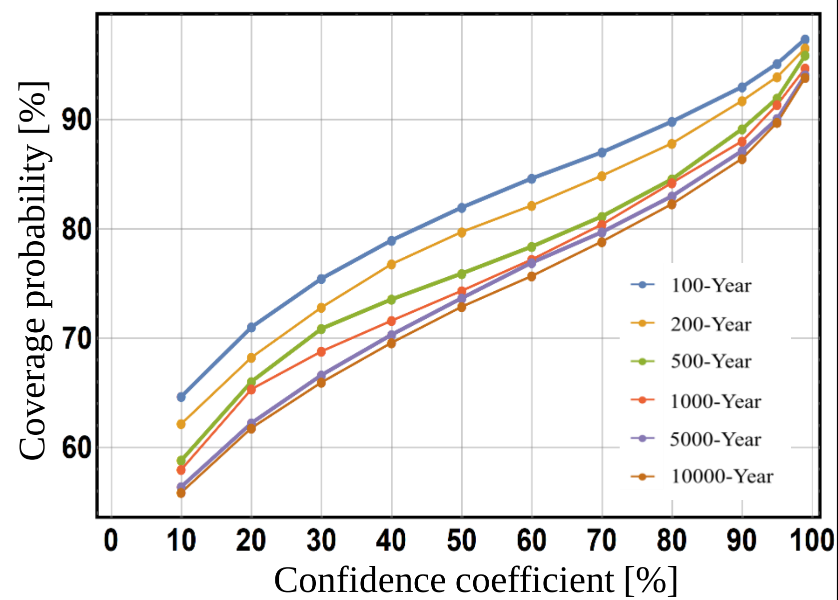
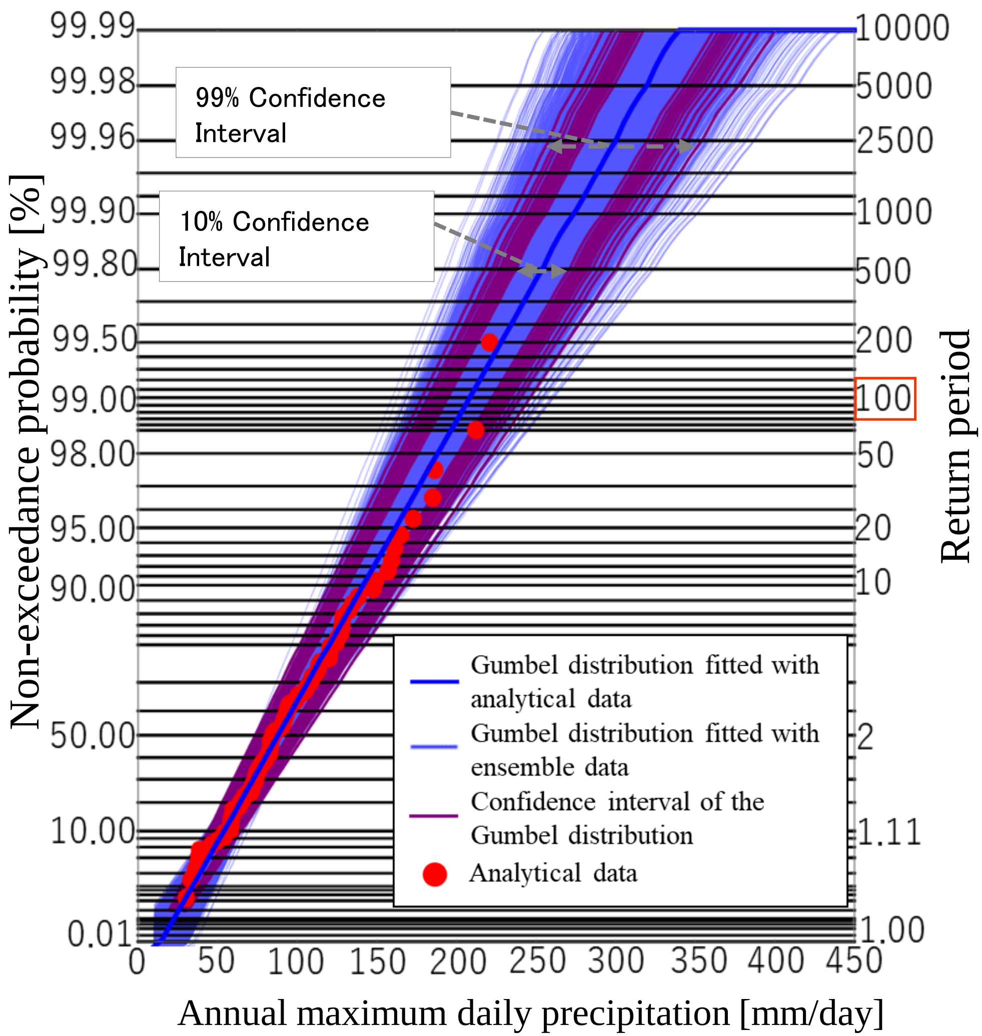


Fig. Relationship between coverage probability and confidence coefficient

Sample size n = 100
Gumbel distribution adopted



Analytical data (n=100) on above probability paper are random numbers according to the Gumbel distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, Gumbel distribution fitted with analytical data and 5000 Gumbel distribution fitted with ensemble data (n=100), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the Gumbel distribution fitted with analytical data

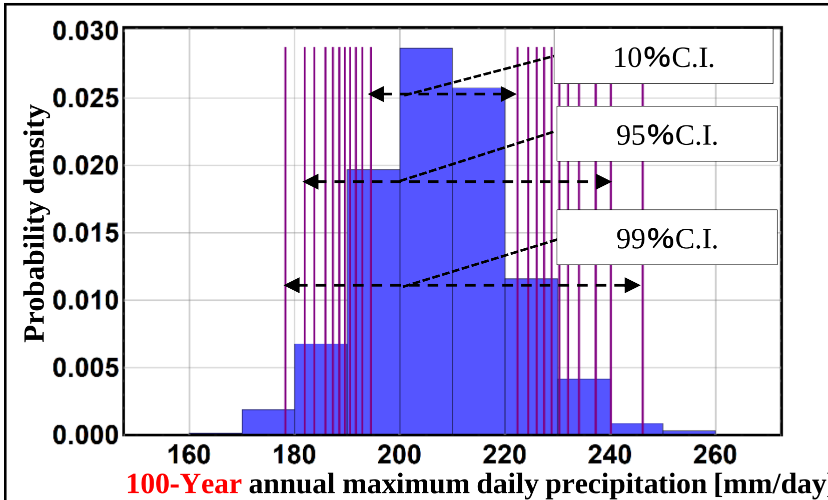


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [194.5, 222.3] = **68.0%**
Coverage probability of 95% C.I. [181.9, 240.1] = **95.4%**
Coverage probability of 99% C.I. [178.2, 246.2] = **97.6%**

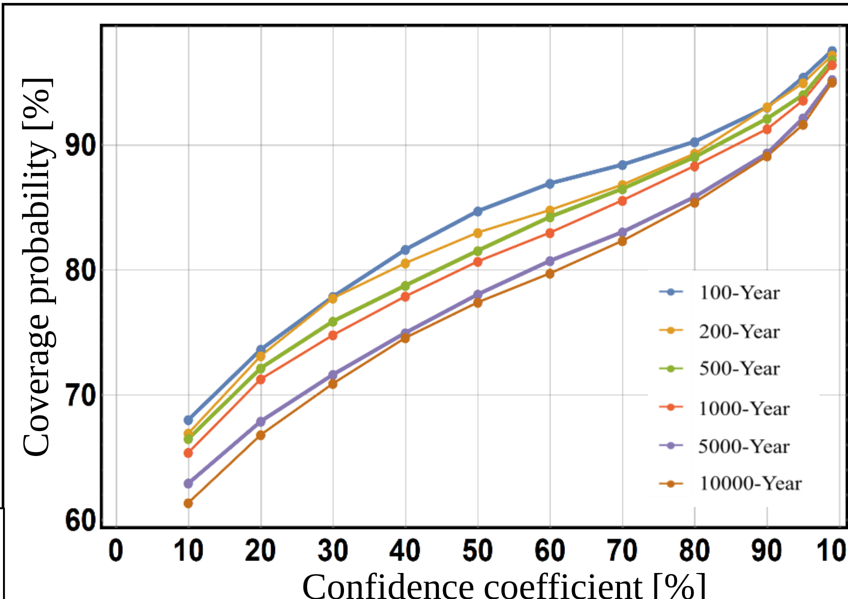
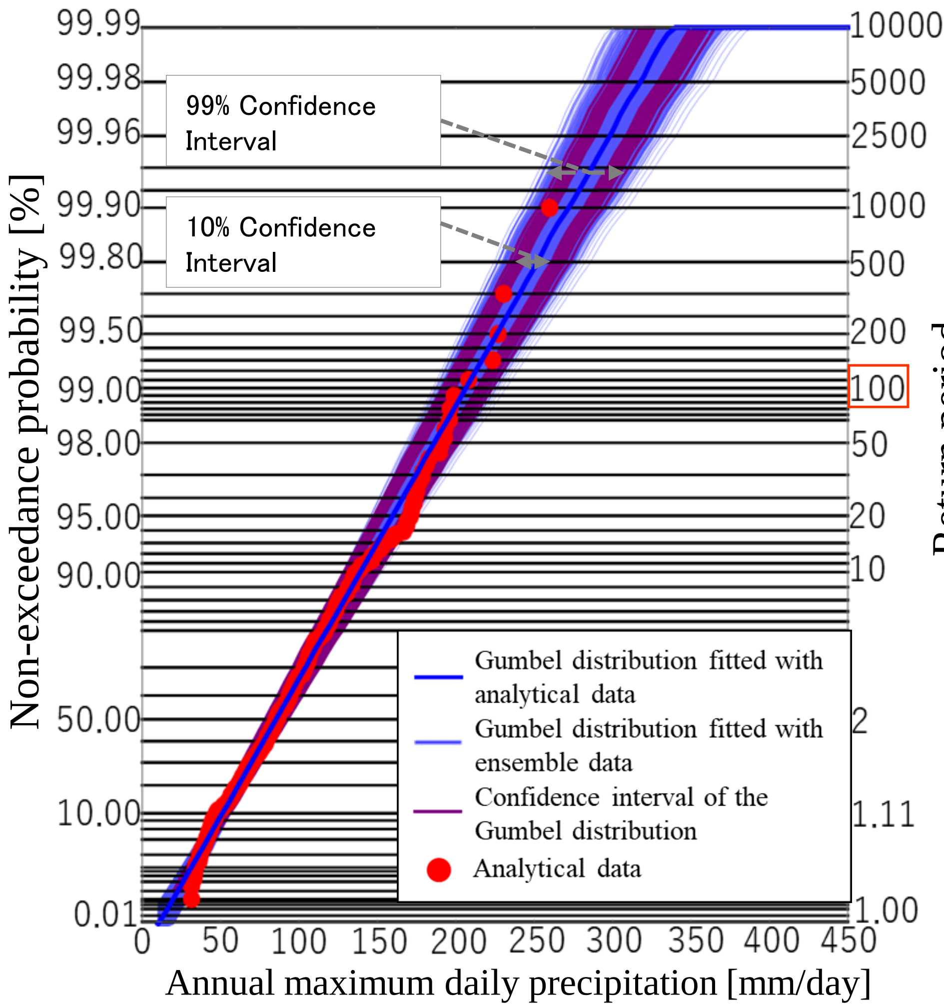


Fig. Relationship between coverage probability and confidence coefficient

Sample size n = 500
Gumbel distribution adopted



Analytical data (n=500) on above probability paper are random numbers according to the Gumbel distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, Gumbel distribution fitted with analytical data and 5000 Gumbel distribution fitted with ensemble data (n=500), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the Gumbel distribution fitted with analytical data

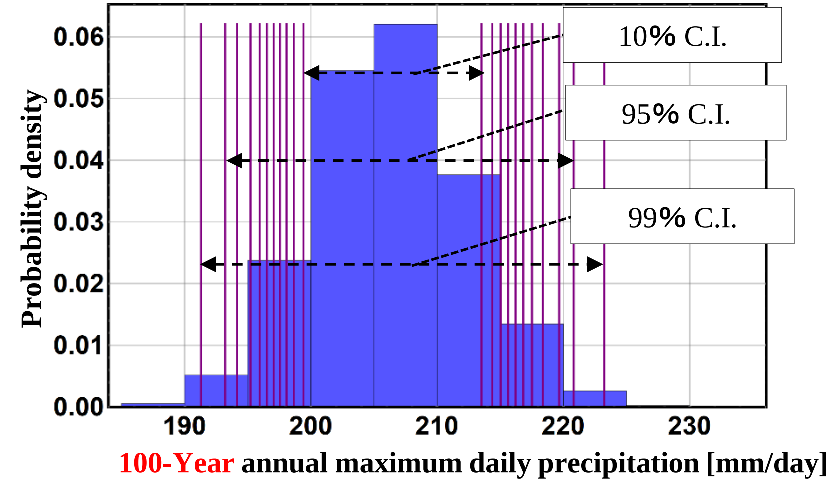


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [199.4, 213.5] = **72.8%**
Coverage probability of 95% C.I. [193.2, 220.8] = **96.8%**
Coverage probability of 99% C.I. [191.3, 223.2] = **98.5%**

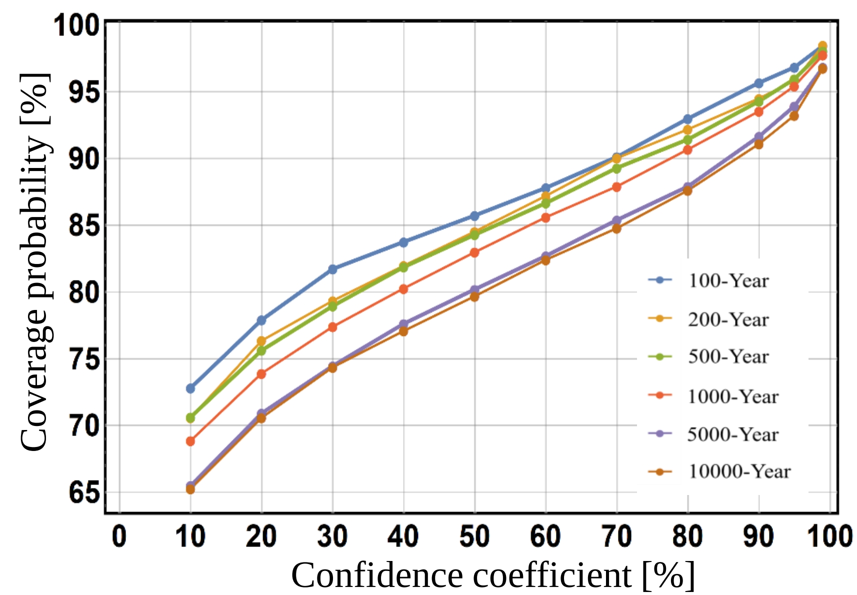
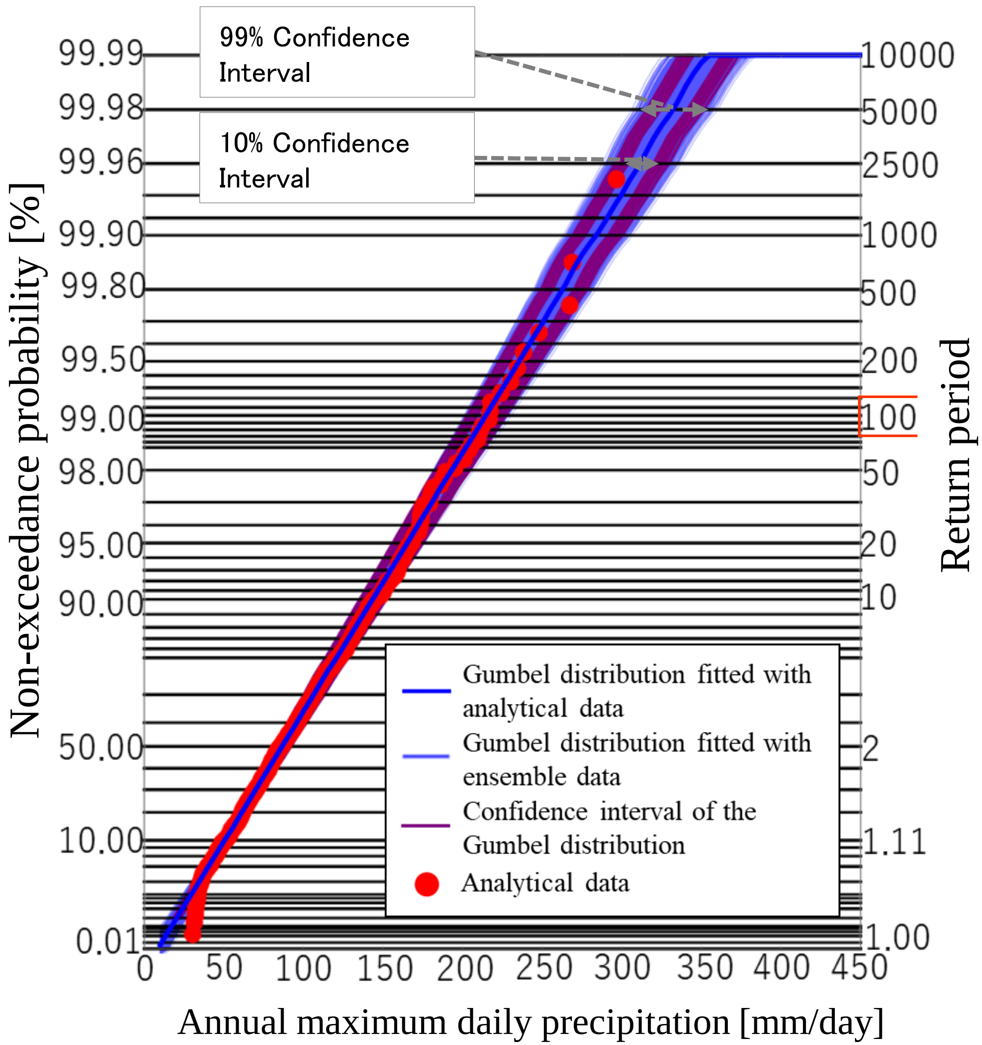


Fig. Relationship between coverage probability and confidence coefficient

Sample size n = 1000
Gumbel distribution adopted



Analytical data (n=1000) on above probability paper are random numbers according to the Gumbel distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, Gumbel distribution fitted with analytical data and 5000 Gumbel distribution fitted with ensemble data (n=1000), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the Gumbel distribution fitted with analytical data

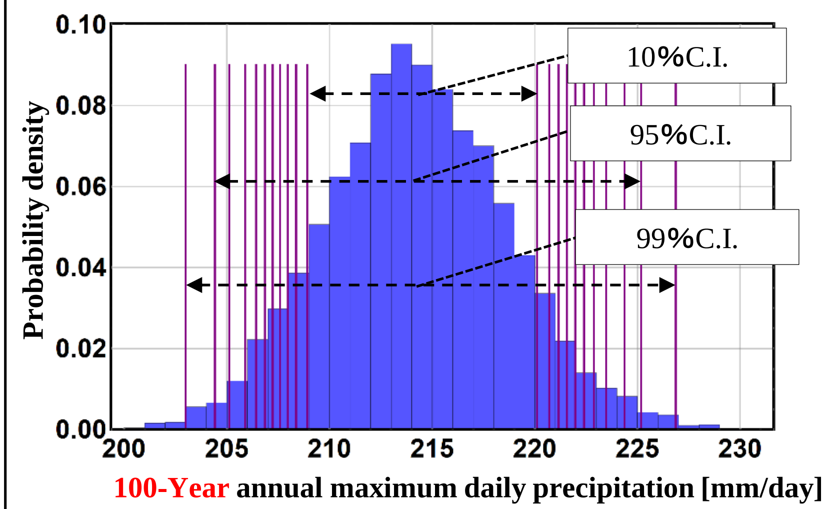


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [208.9, 220.1] = **79.0%**
Coverage probability of 95% C.I. [204.4, 225.2] = **97.8%**
Coverage probability of 99% C.I. [203.0, 226.9] = **99.3%**

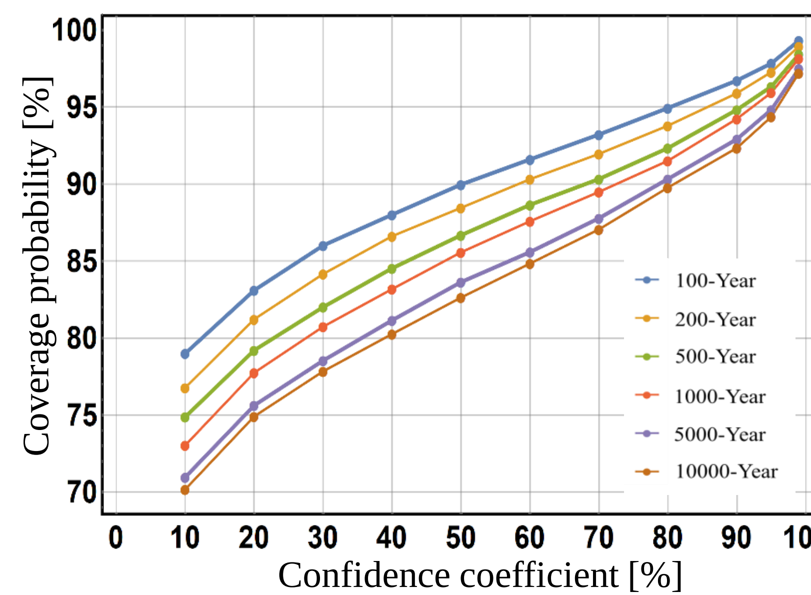
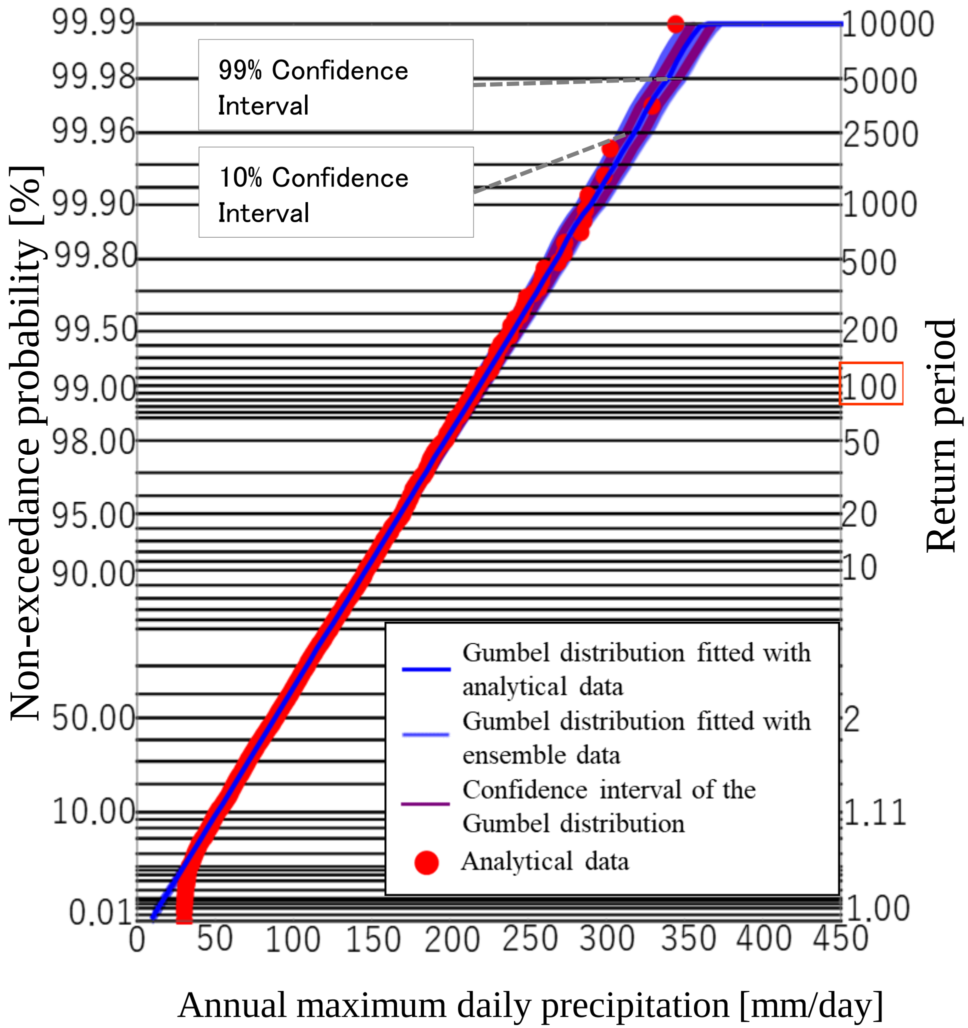


Fig. Relationship between coverage probability and confidence coefficient

Sample size n = 5000
Gumbel distribution adopted



Analytical data (n=5000) on above probability paper are random numbers according to the Gumbel distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, Gumbel distribution fitted with analytical data and 5000 Gumbel distribution fitted with ensemble data (n=5000), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the Gumbel distribution fitted with analytical data

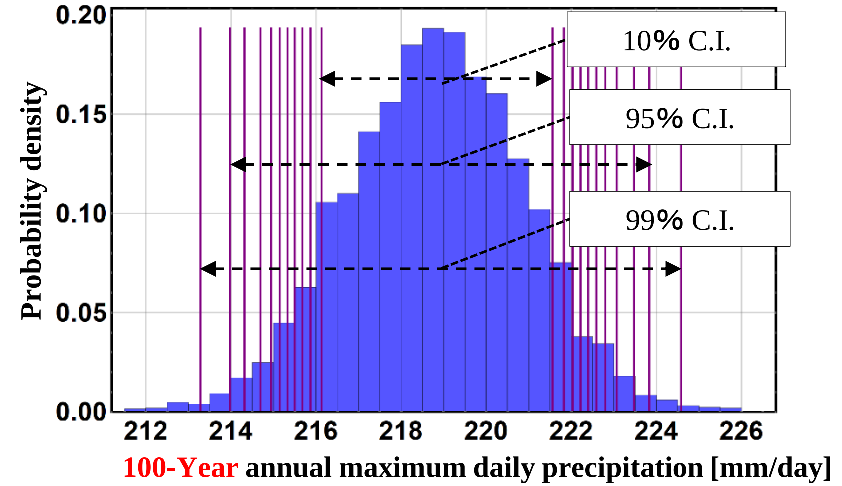


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [211.5, 216.7] = **81.1%**
Coverage probability of 95% C.I. [209.4, 218.9] = **98.1%**
Coverage probability of 99% C.I. [208.7, 219.7] = **99.1%**

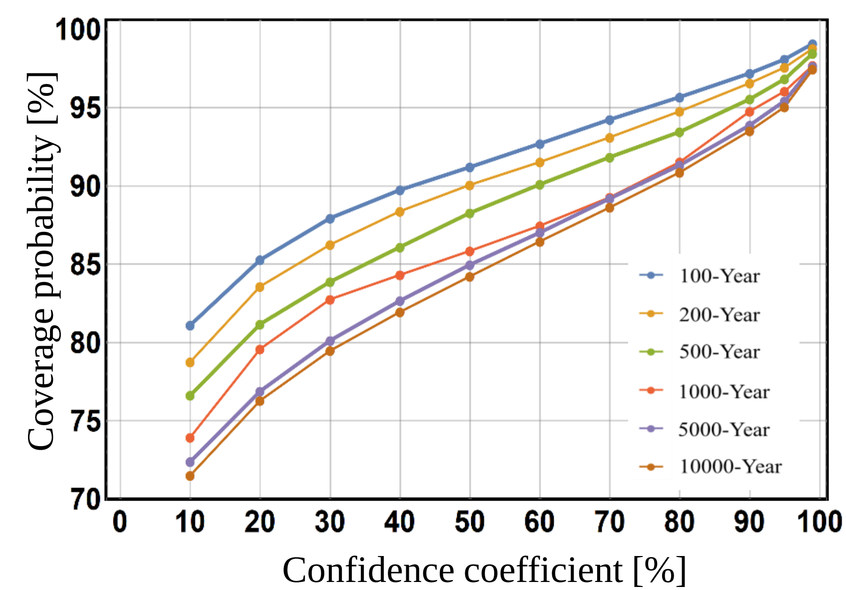
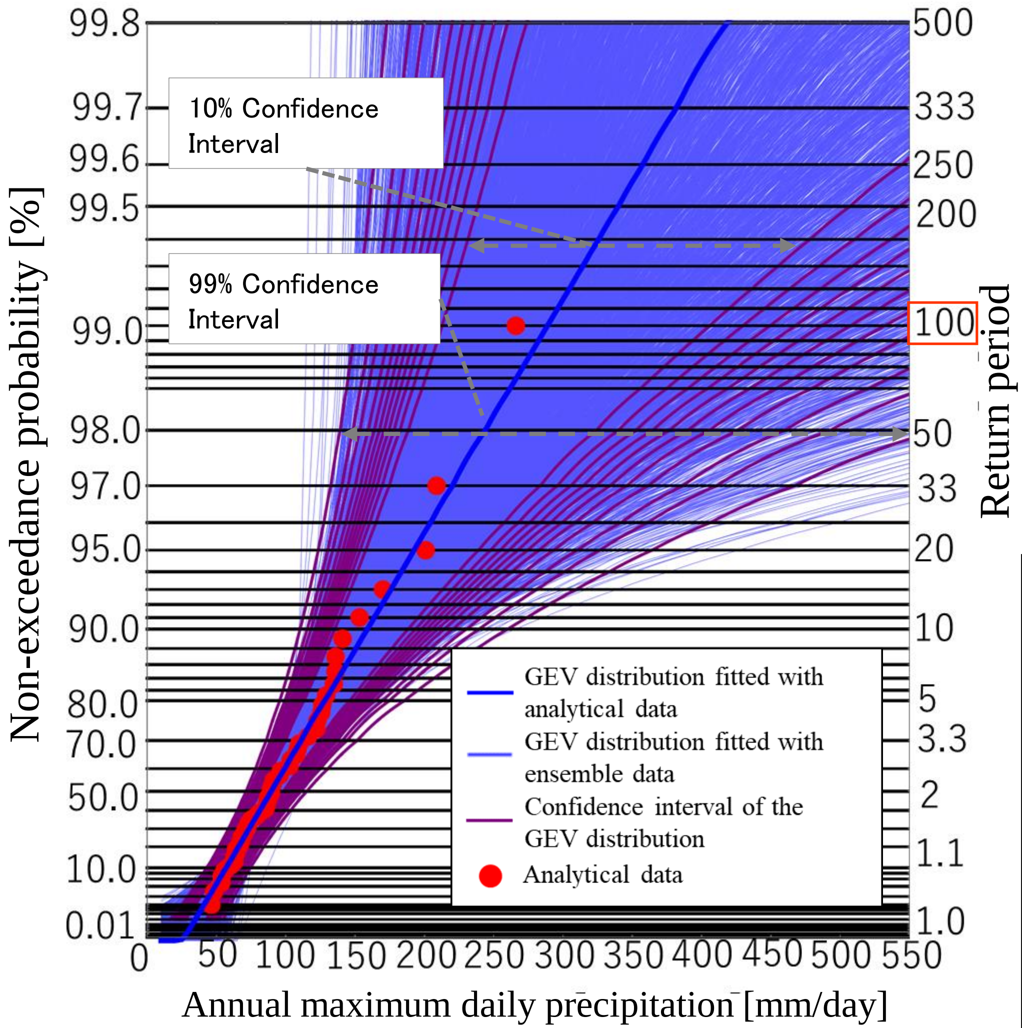


Fig. Relationship between coverage probability and confidence coefficient

Sample size *n* = 50
G.E.V. distribution adopted



Analytical data (n=50) on above probability paper are random numbers according to the GEV distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, GEV distribution fitted with analytical data and 5000 GEV distribution fitted with ensemble data (n=50), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the GEV distribution fitted with analytical data

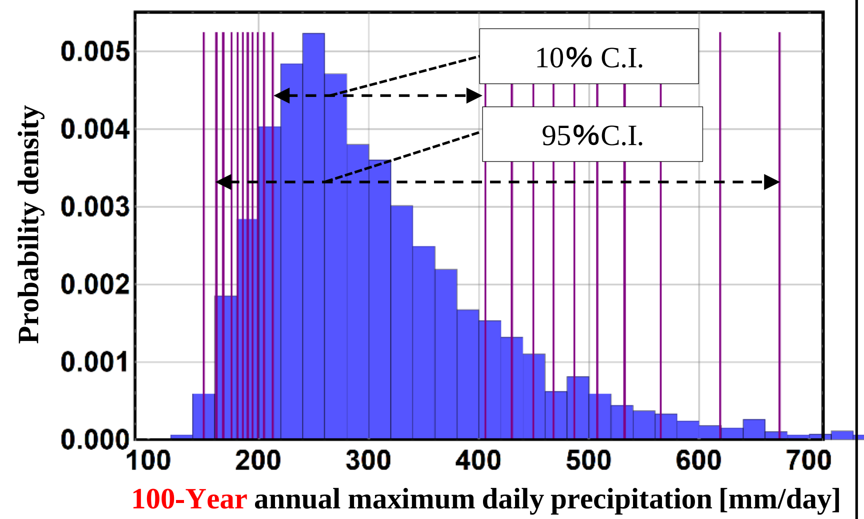


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [212.8, 405.8] = **66.9%**
Coverage probability of 95% C.I. [161.5, 673.0] = **96.2%**
Coverage probability of 99% C.I. [150.1 803.5] = **98.2%**

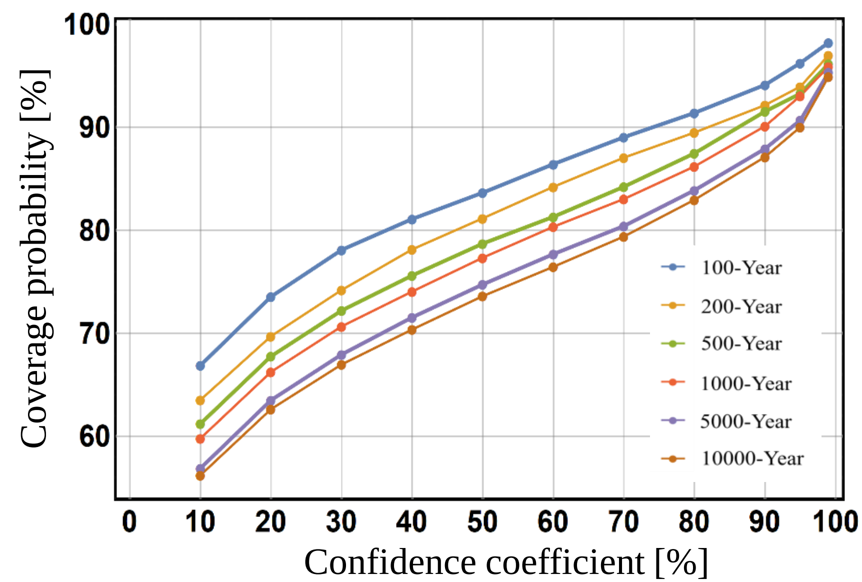
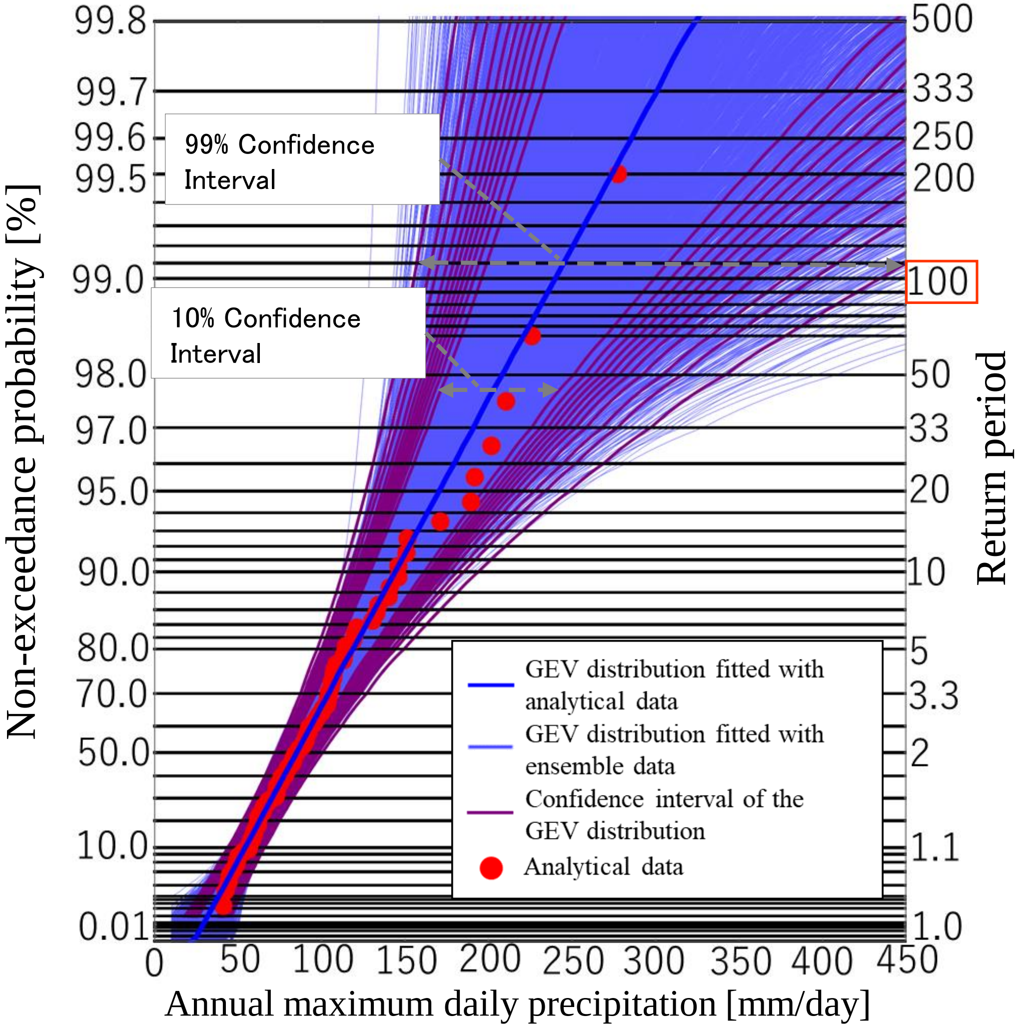


Fig. Relationship between coverage probability and confidence coefficient

Sample size n = 100
G.E.V. distribution adopted



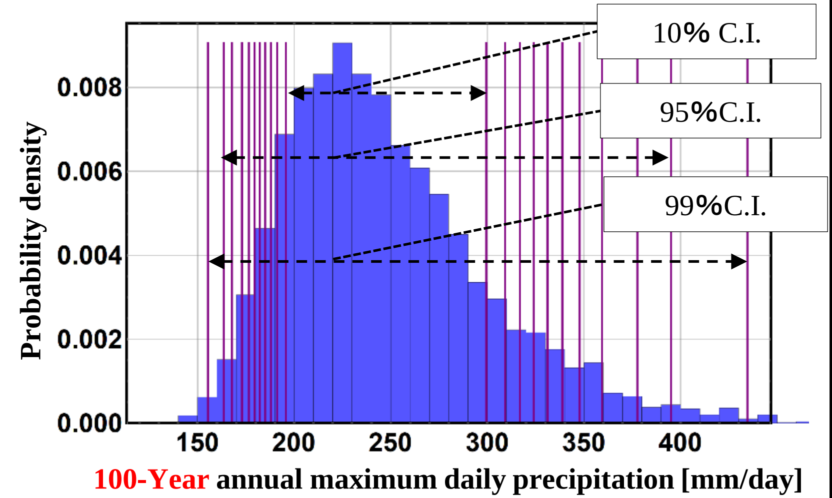


Fig. 100-Year quantile distribution and confidence interval

Coverage probability of 10% C.I. [195.8, 299.5] = **70.2%**

Coverage probability of 95% C.I. [163.5, 395.1] = **96.9%**

Coverage probability of 99% C.I. [155.3, 434.6] = **98.9%**

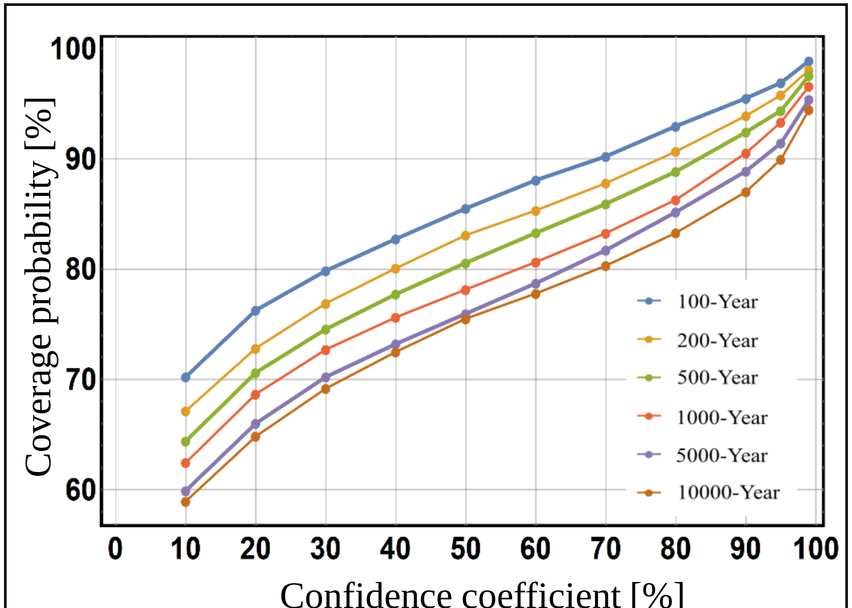
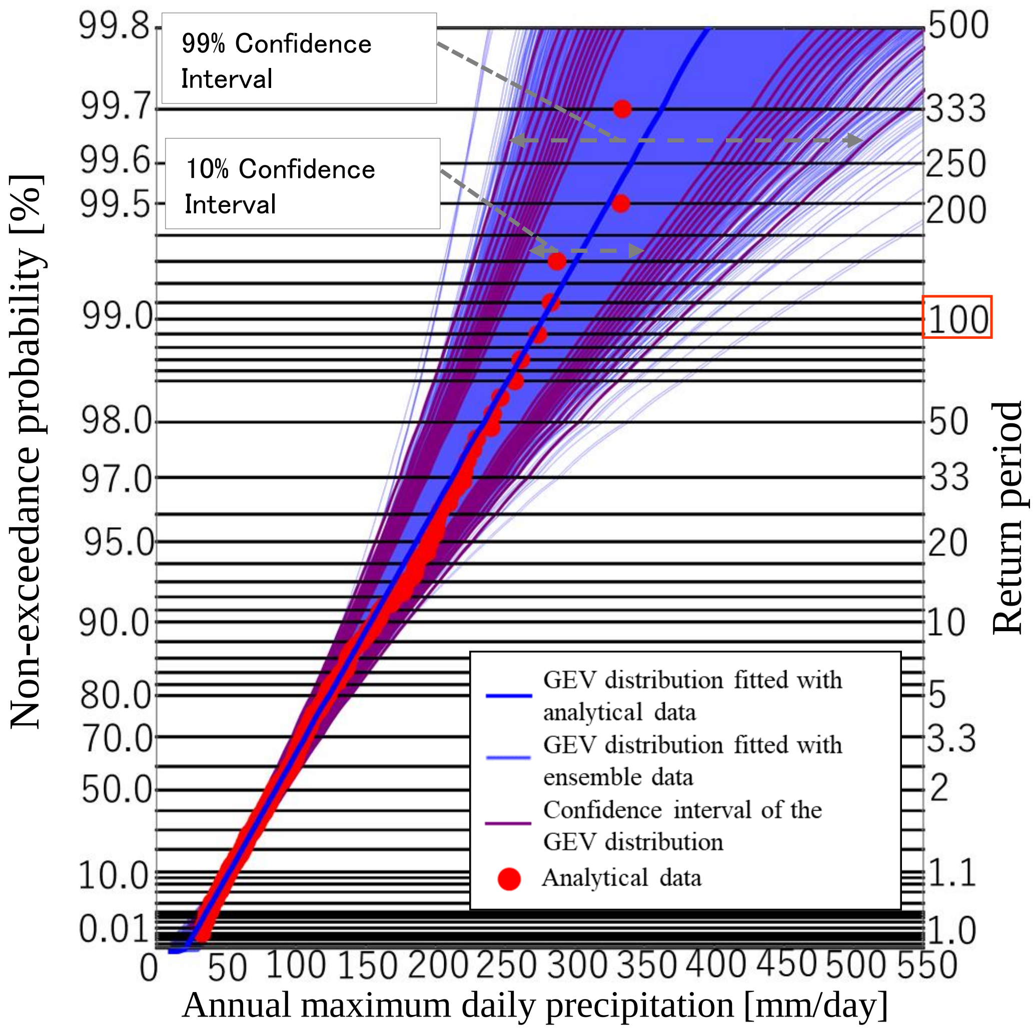


Fig. Relationship between coverage probability and confidence coefficient

Analytical data (n=100) on above probability paper are random numbers according to the GEV distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, GEV distribution fitted with analytical data and 5000 GEV distribution fitted with ensemble data (n=100), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the GEV distribution fitted with analytical data

Sample size n = 500
G.E.V. distribution adopted



Analytical data (n=500) on above probability paper are random numbers according to the GEV distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, GEV distribution fitted with analytical data and 5000 GEV distribution fitted with ensemble data (n=500), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the GEV distribution fitted with analytical data

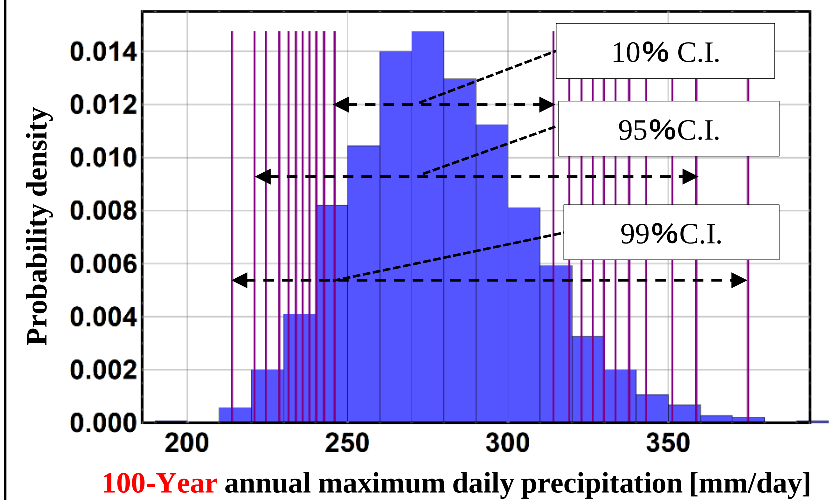


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [245.9, 314.1] = **77.3%**
Coverage probability of 95% C.I. [221.0, 358.7] = **98.4%**
Coverage probability of 99% C.I. [213.9, 374.8] = **99.4%**

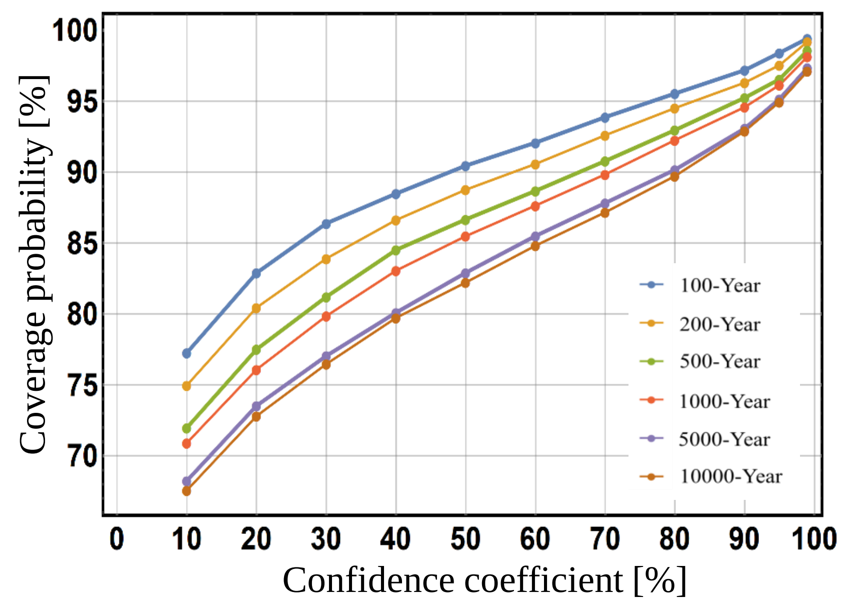
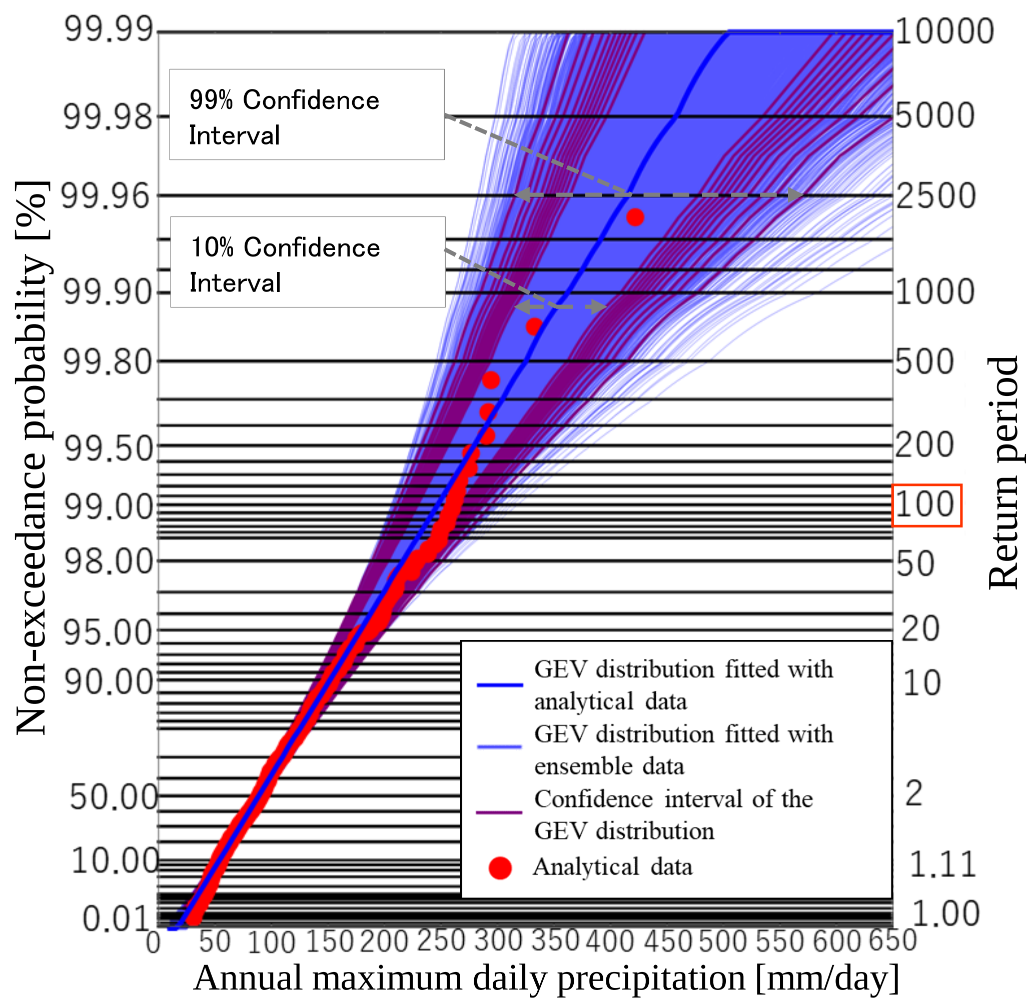


Fig. Relationship between coverage probability and confidence coefficient

Sample size *n* = 1000
G.E.V. distribution adopted



Analytical data (n=1000) on above probability paper are random numbers according to the GEV distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, GEV distribution fitted with analytical data and 5000 GEV distribution fitted with ensemble data (n=1000), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the GEV distribution fitted with analytical data

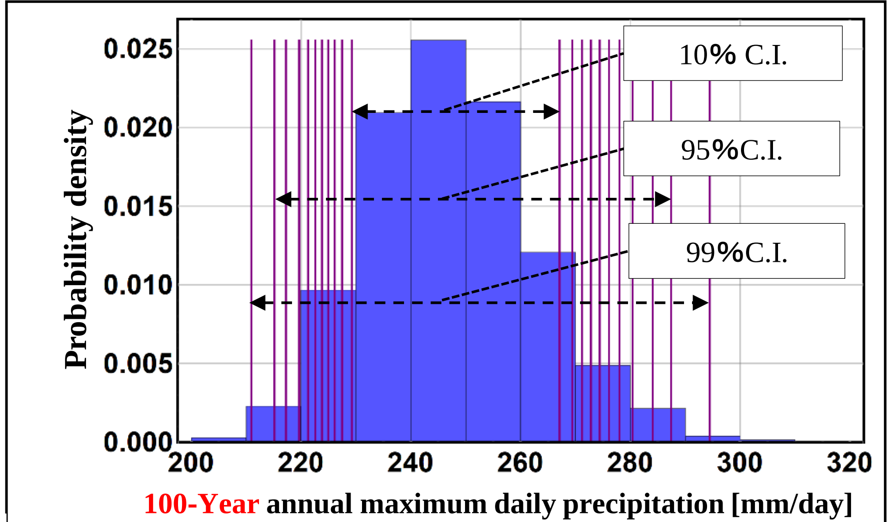


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [229.2, 267.1] = **77.4%**
Coverage probability of 95% C.I. [215.1, 287.4] = **97.4%**
Coverage probability of 99% C.I. [210.9, 294.5] = **99.1%**

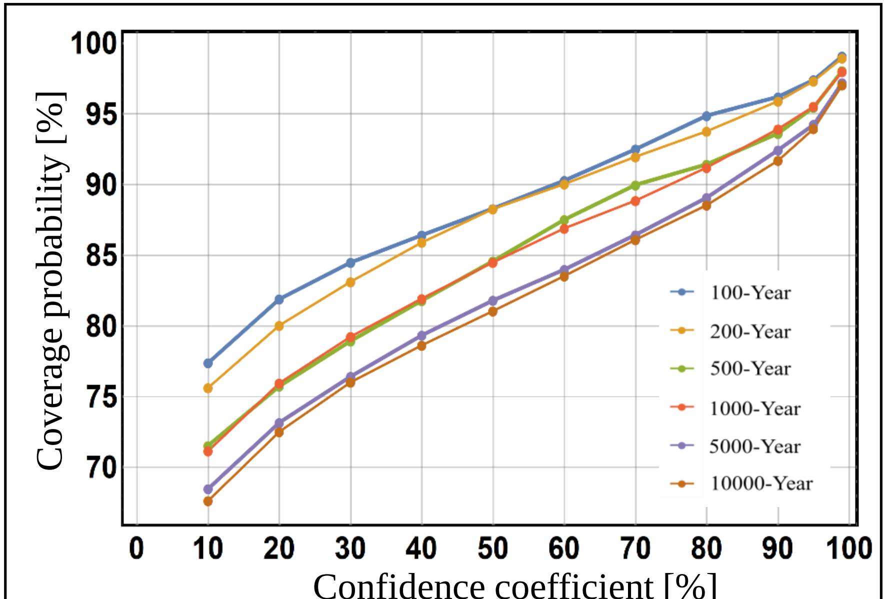
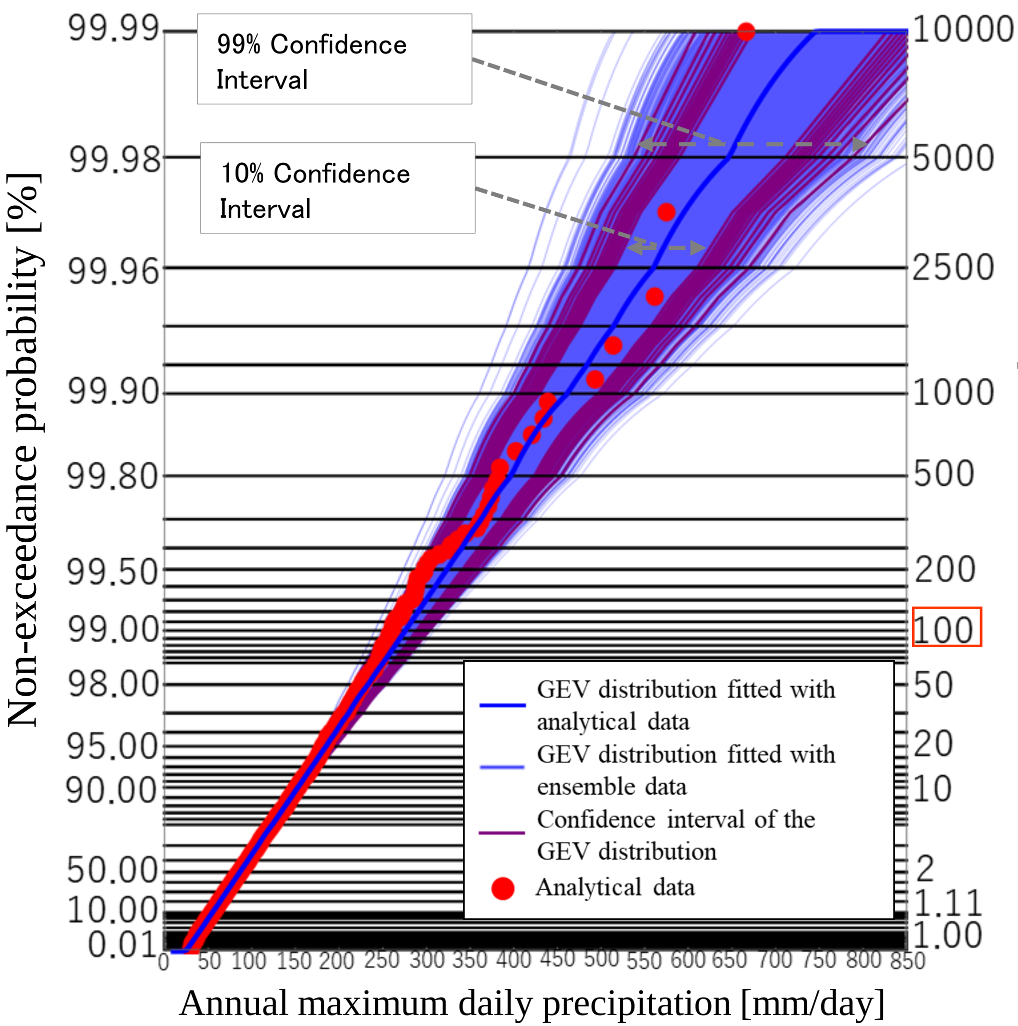


Fig. Relationship between coverage probability and confidence coefficient

Sample size *n* = 5000
G.E.V. distribution adopted



Analytical data (n=5000) on above probability paper are random numbers according to the GEV distribution fitted with the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tonegawa River system. Also, GEV distribution fitted with analytical data and 5000 GEV distribution fitted with ensemble data (n=5000), 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% were written in this probability paper. Ensemble data is composed of random numbers according to the GEV distribution fitted with analytical data

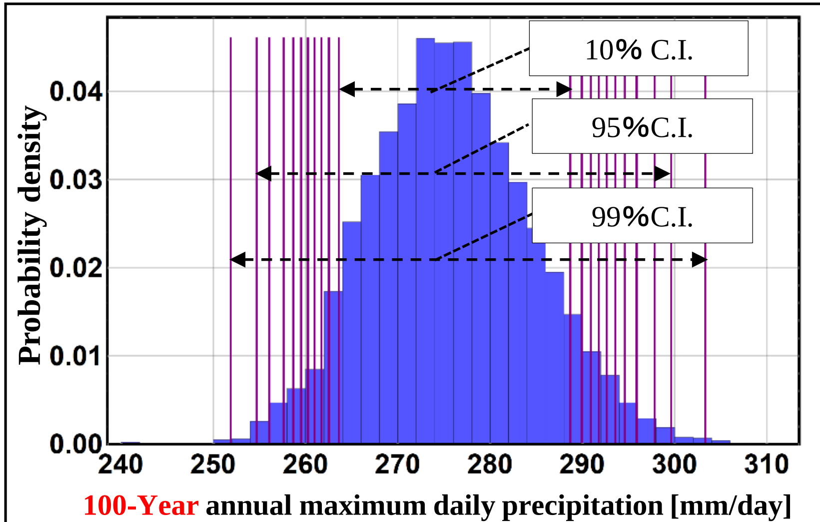


Fig. **100-Year quantile distribution and confidence interval**

Coverage probability of 10% C.I. [263.6, 288.7] = **84.6%**

Coverage probability of 95% C.I. [254.7, 299.6] = **99.0%**

Coverage probability of 99% C.I. [251.9, 303.3] = **99.7%**

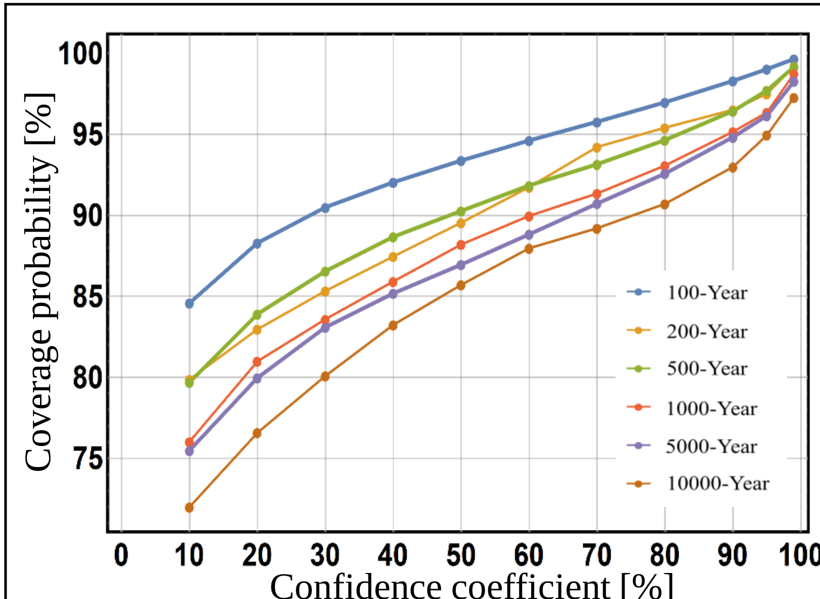
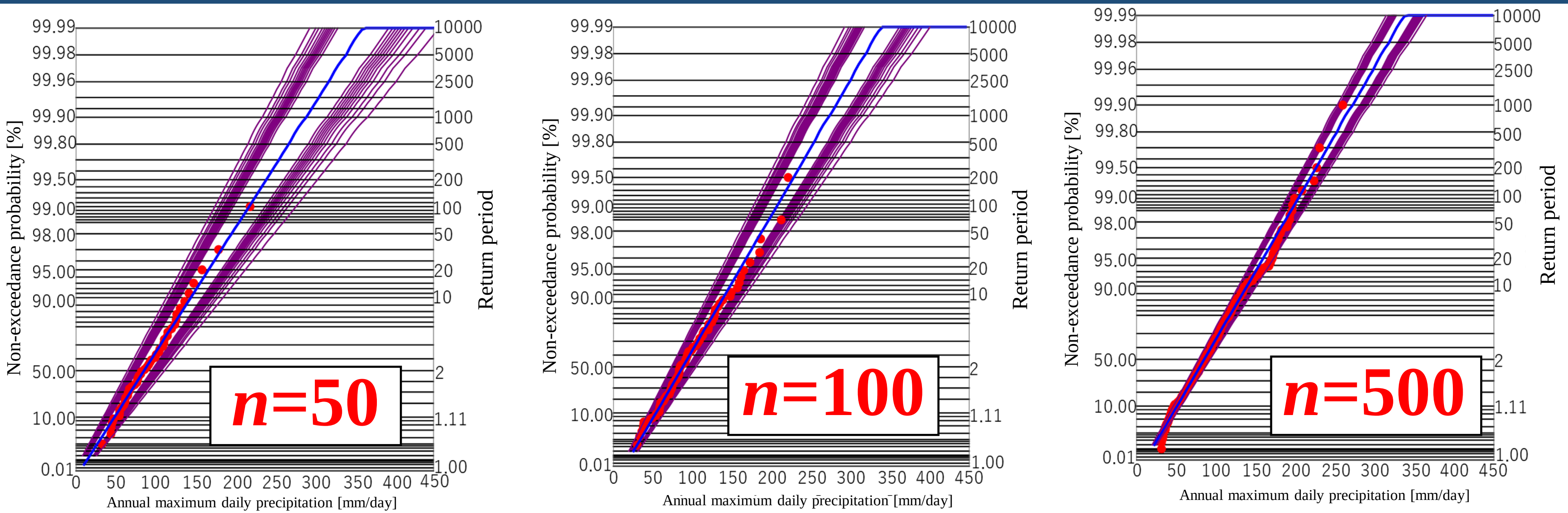
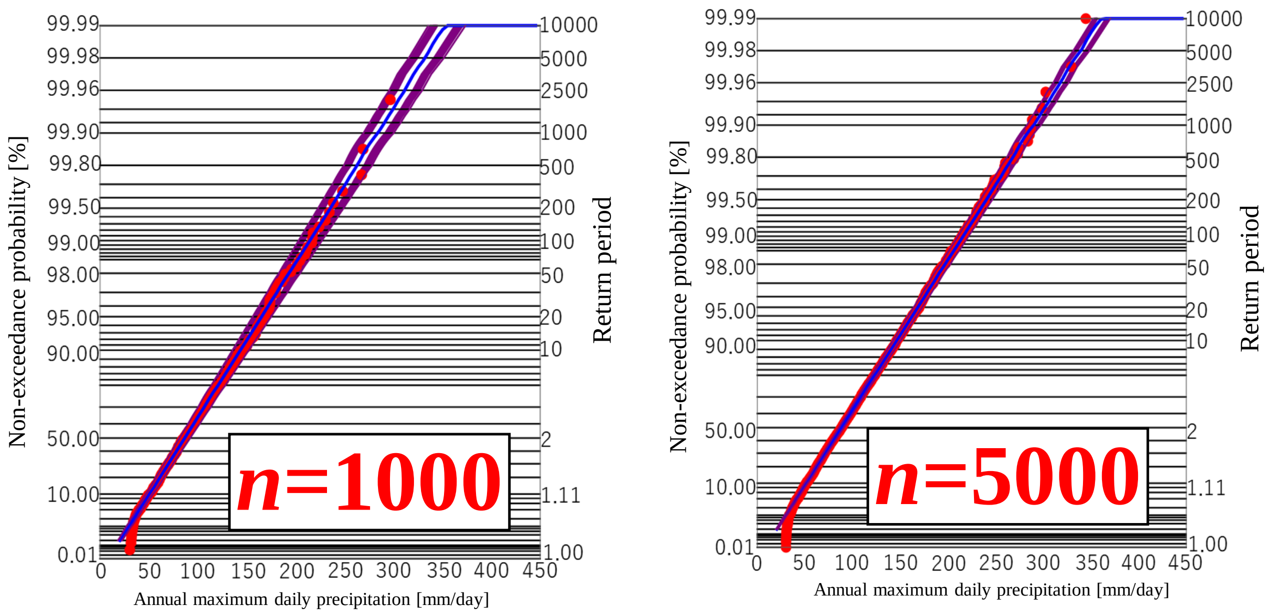


Fig. Relationship between coverage probability and confidence coefficient

Relationship between sample size and confidence interval (Gumbel)





As the number of data (sampling number) increases, the confidence interval narrows, and the reliability of estimation improves.

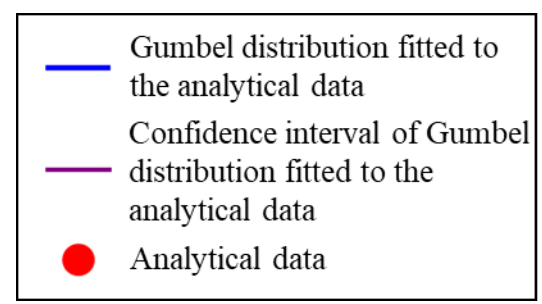
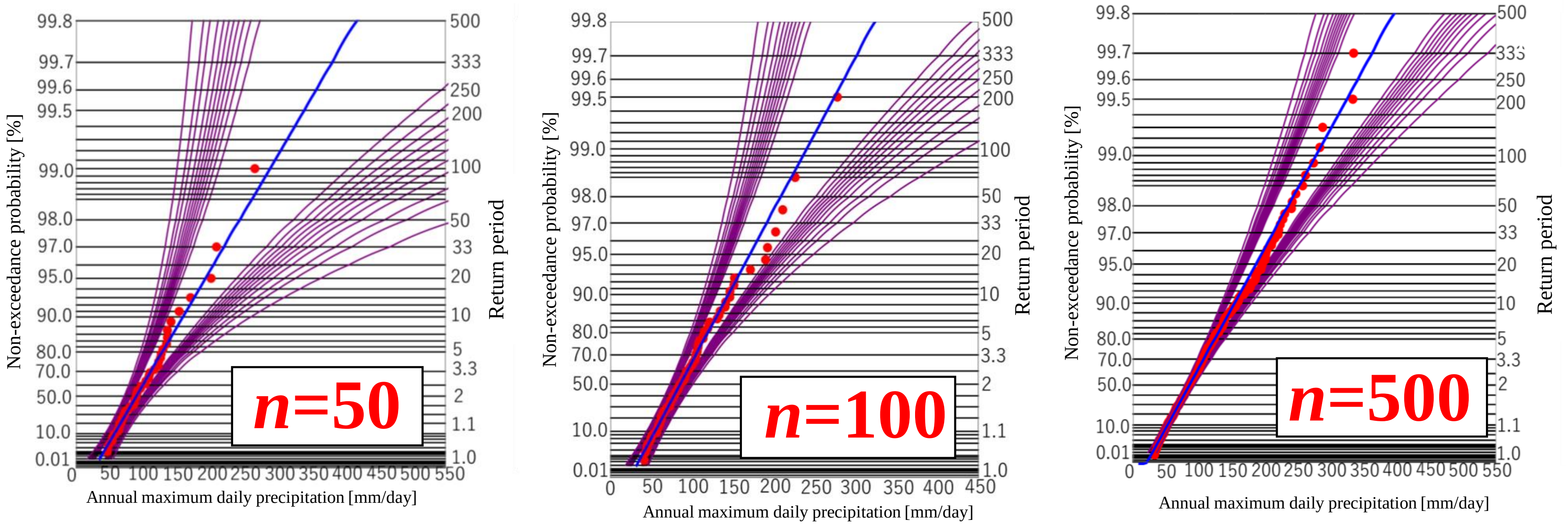
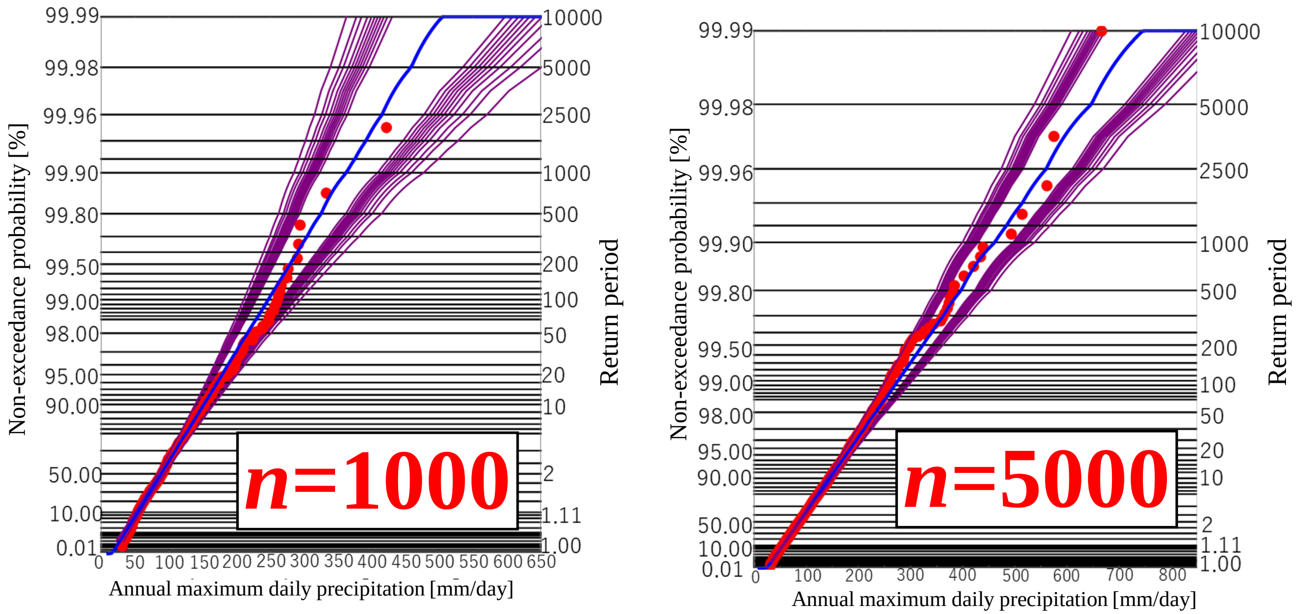


Fig. Relationship between sampling number and confidence interval in the case of adopting Gumbel distribution

Analytical data (red dot) in each probability paper is a random number according to the Gumbel distribution fitted to the observed data of the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tone River system. In each probability paper, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% confidence intervals were written. Here, *n* represents the sampling number (total number of analysis data).

Relationship between sample size and confidence interval (GEV)





As the number of data (sampling number) increases, the confidence interval narrows, and the reliability of estimation improves.

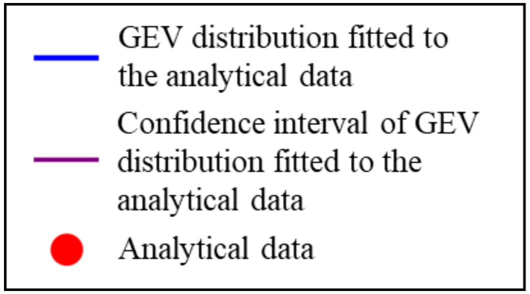


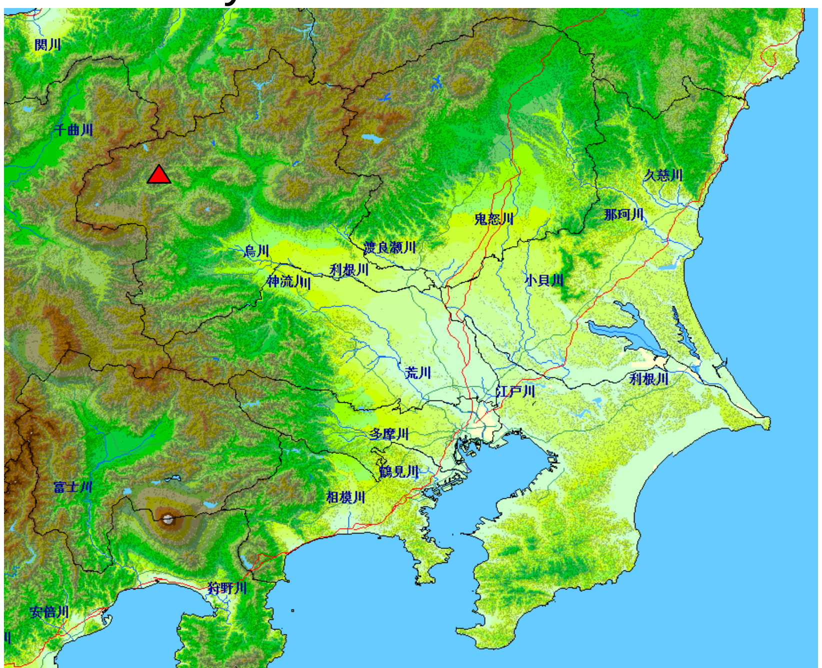
Fig. Relationship between sampling number and confidence interval in the case of adopting GEV distribution

Analytical data (red dot) in each probability paper is a random number according to the GEV distribution fitted to the observed data of the annual maximum daily precipitation for 54 years at the Yattajima Observatory of the Tone River system. In each probability paper, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 99% confidence intervals were written. Here, *n* represents the sampling number (total number of analysis data).

Occurrence characteristic of extreme rainfall in Japan

Periodicity of extreme rain fall in mountainous area

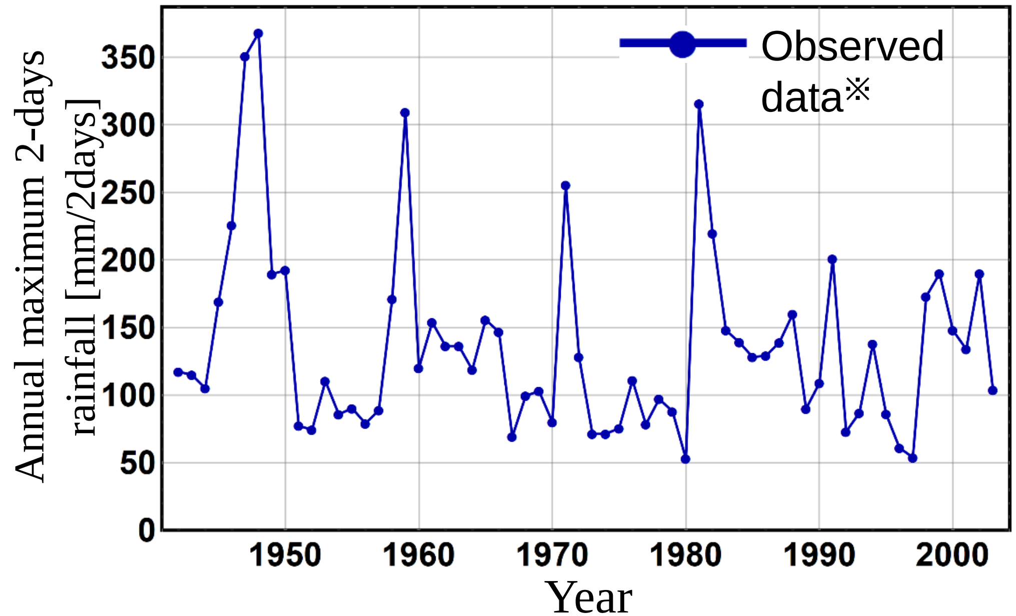
▲: Nakanojou observatory of Agatumagawa river in Tonegawa river system（Elevation: 351m）



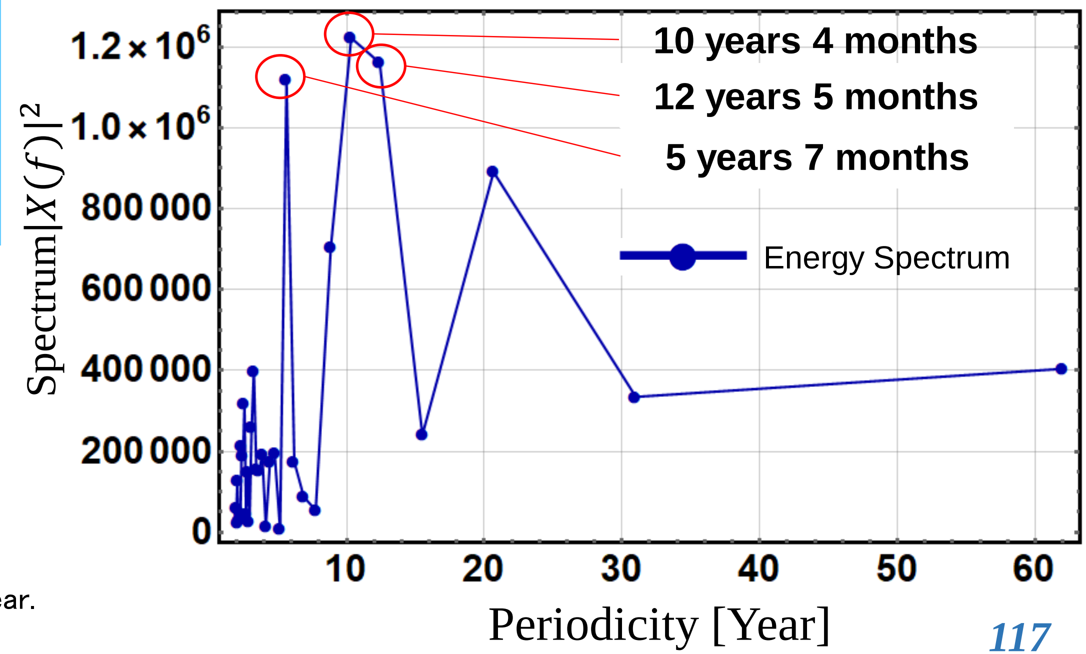
It is seen that 10 year cycle of annual maximum rainfall exists in mountainous area.

※observed data of annual maximum 2-days rainfall at Nakanojou observatory from 1942 to 2003. Missing value is interpolated by average value of data of before and after year. Data of 1962 and 1963 are missing.

Annual maximum 2-days rainfall

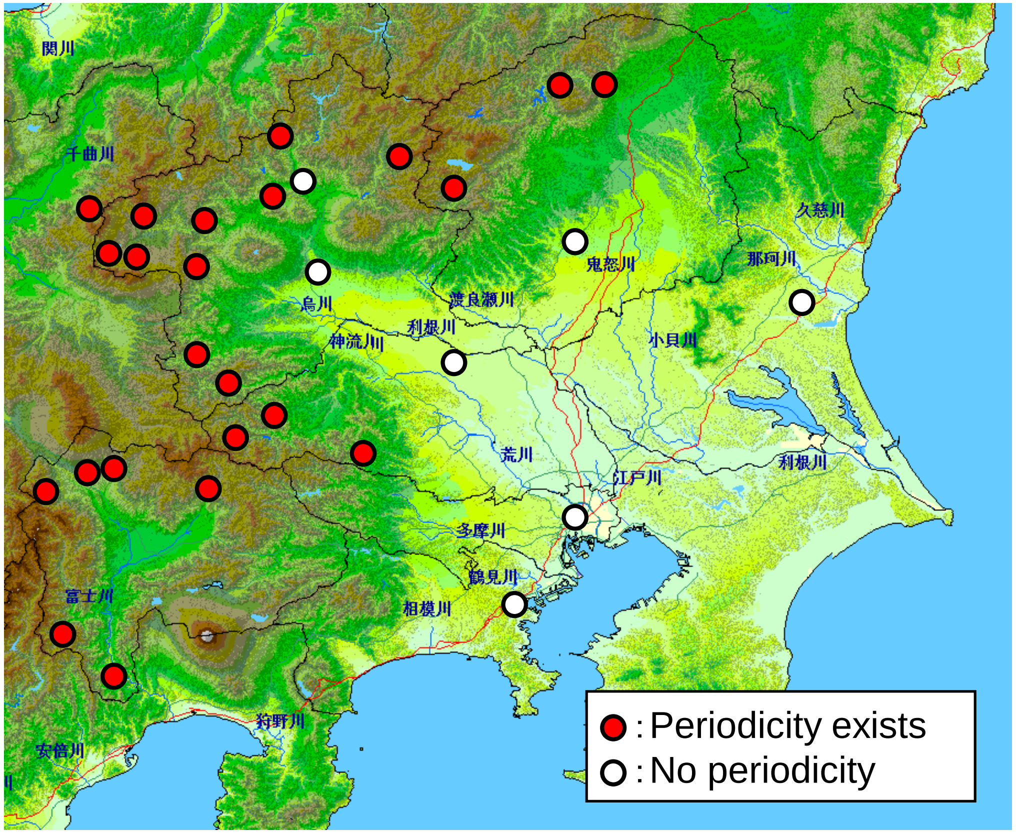


Energy Spectrum

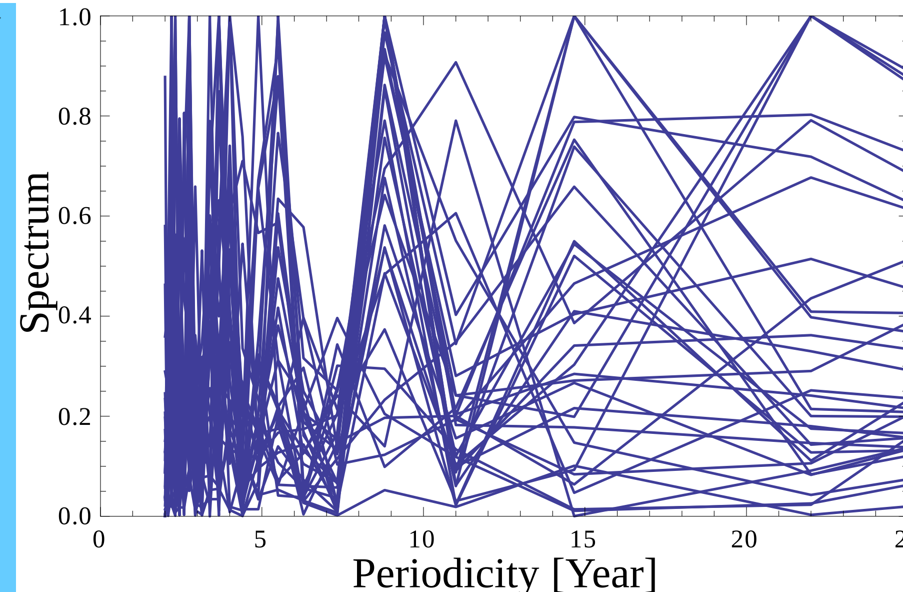


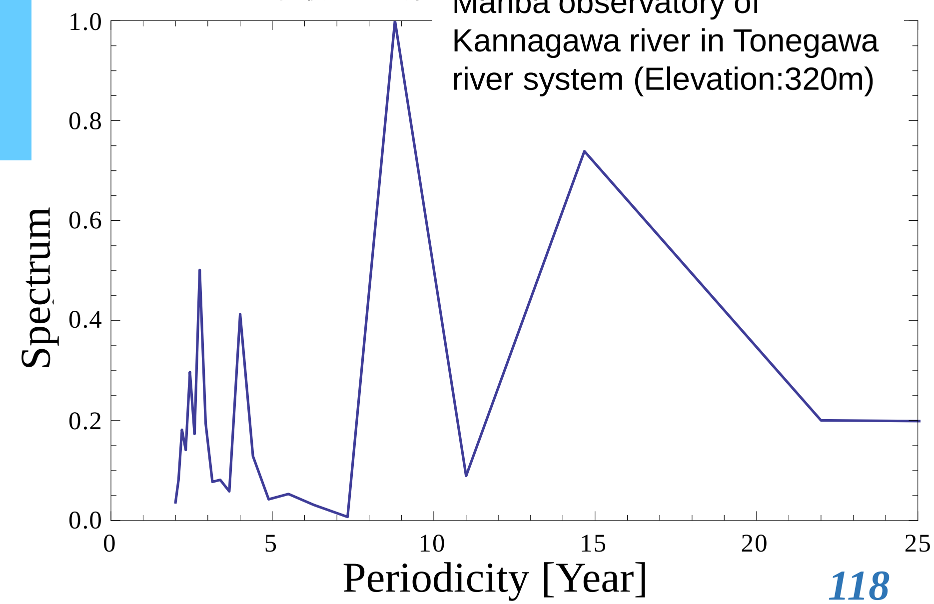
Periodicity of extreme rainfall in Kanto area

Total number of data：44（1960～2003 [year]）



Spectrum of annual maximum 3-days rainfall





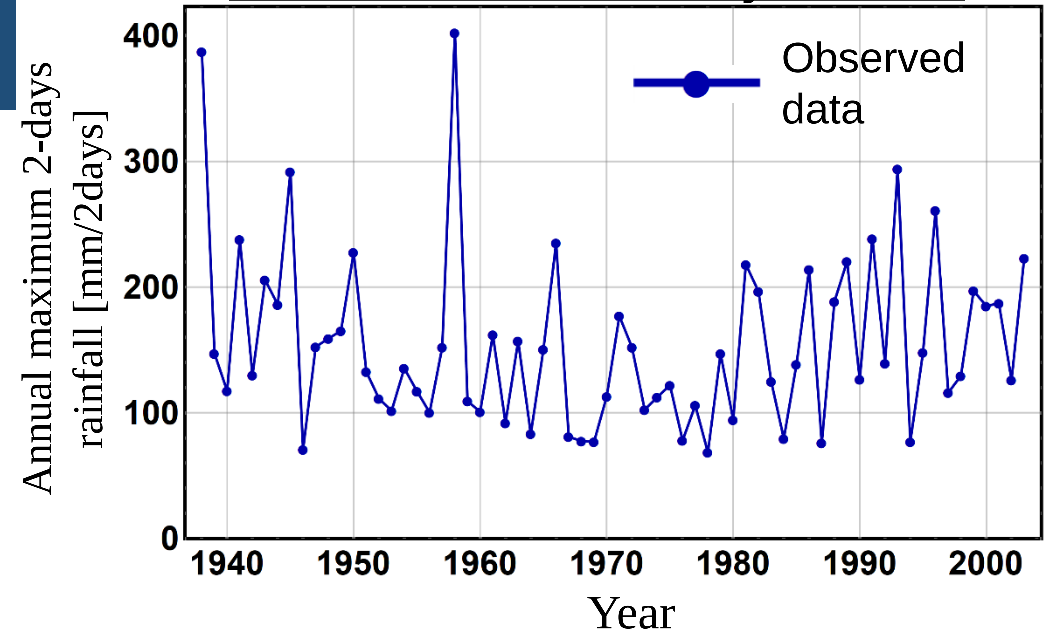
- In mountainous area of Kanto area, there is about **10 years periodicity** of annual maximum 3-days rainfall.
- In plain area of Kanto area, there is no periodicity of annual maximum 3-days rainfall.

Periodicity of extreme rain fall in plain area

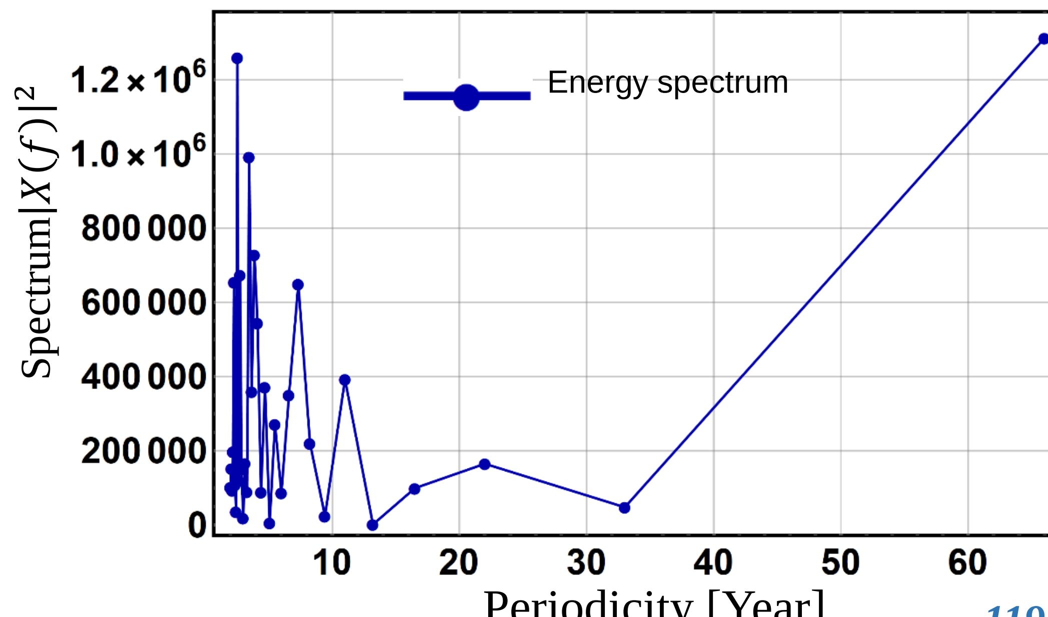
▲ : Ootemachi observatory in Tokyo (Elevation : 6m)

There is no 10 year cycle of annual maximum rainfall in plain area.

Annual maximum 2-days rainfall

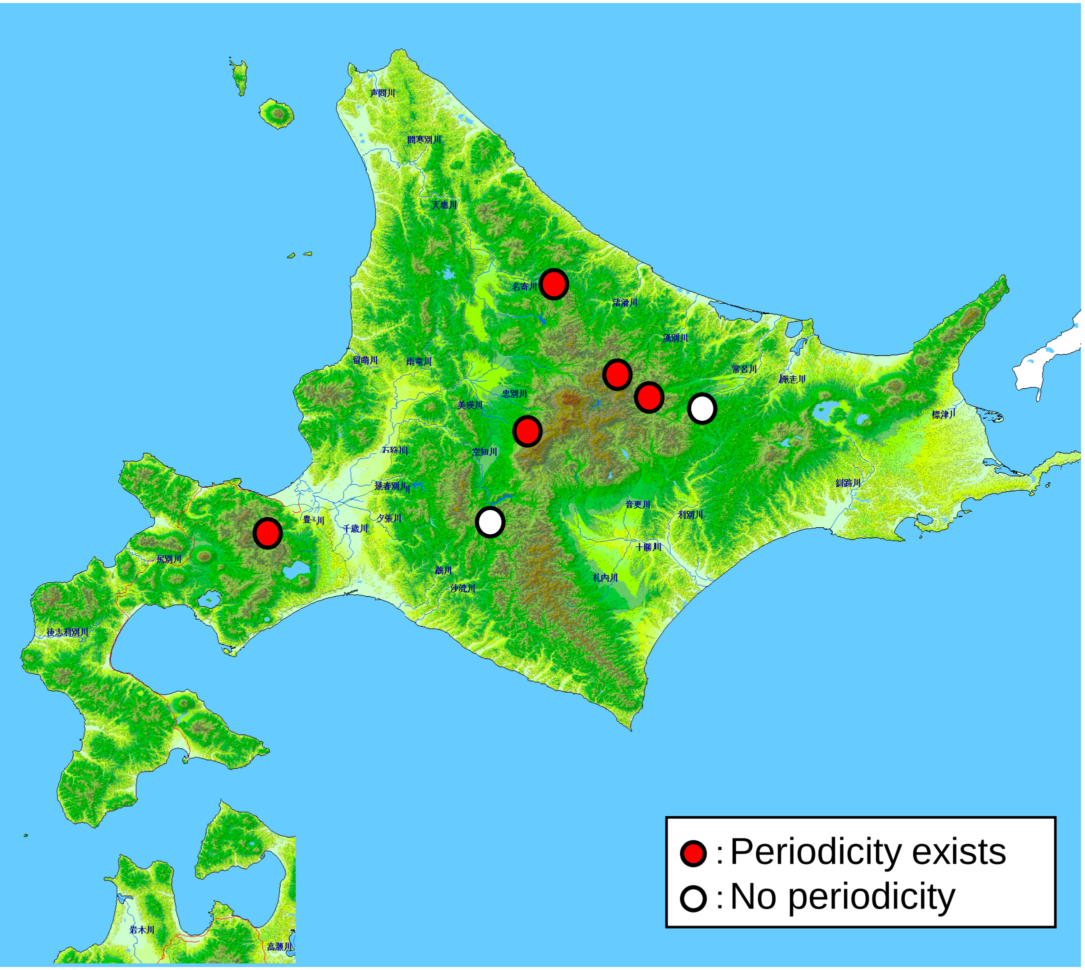


Energy Spectrum

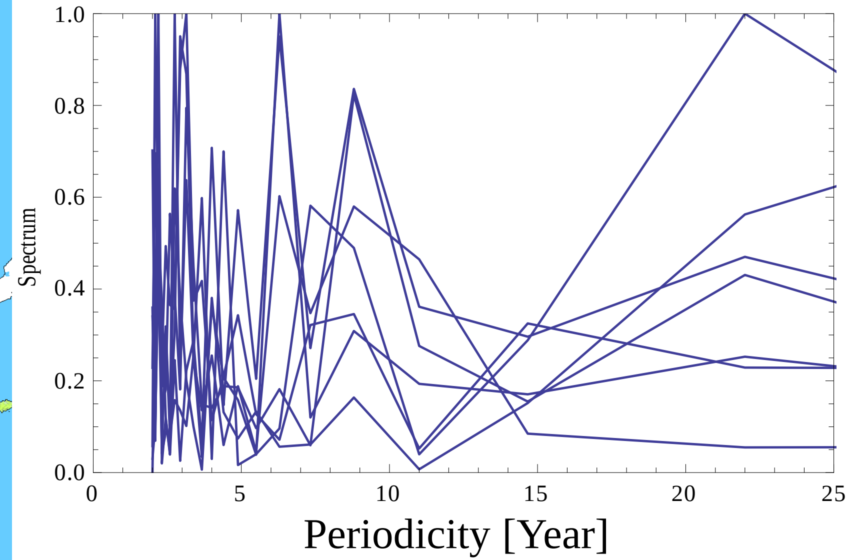


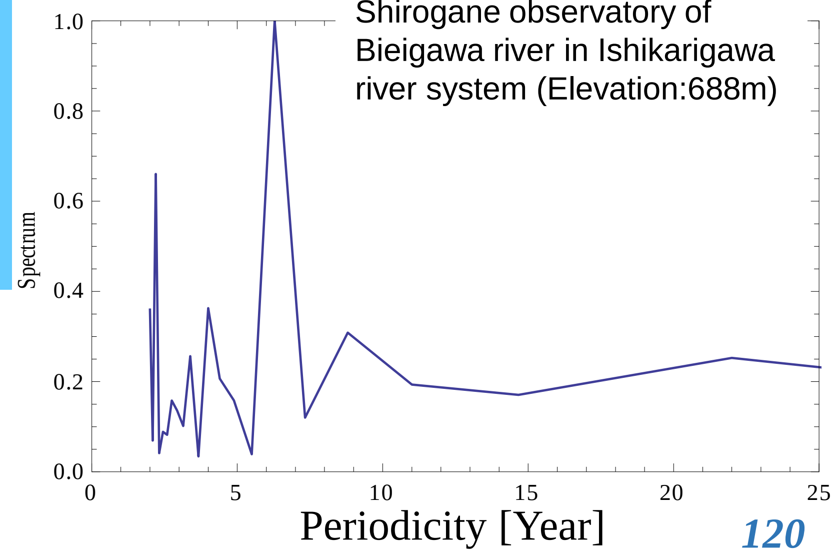
Periodicity of extreme rainfall in Hokkaido area

Total number of data：44（1960～2003 [year]）



Spectrum of annual maximum 3-days rainfall

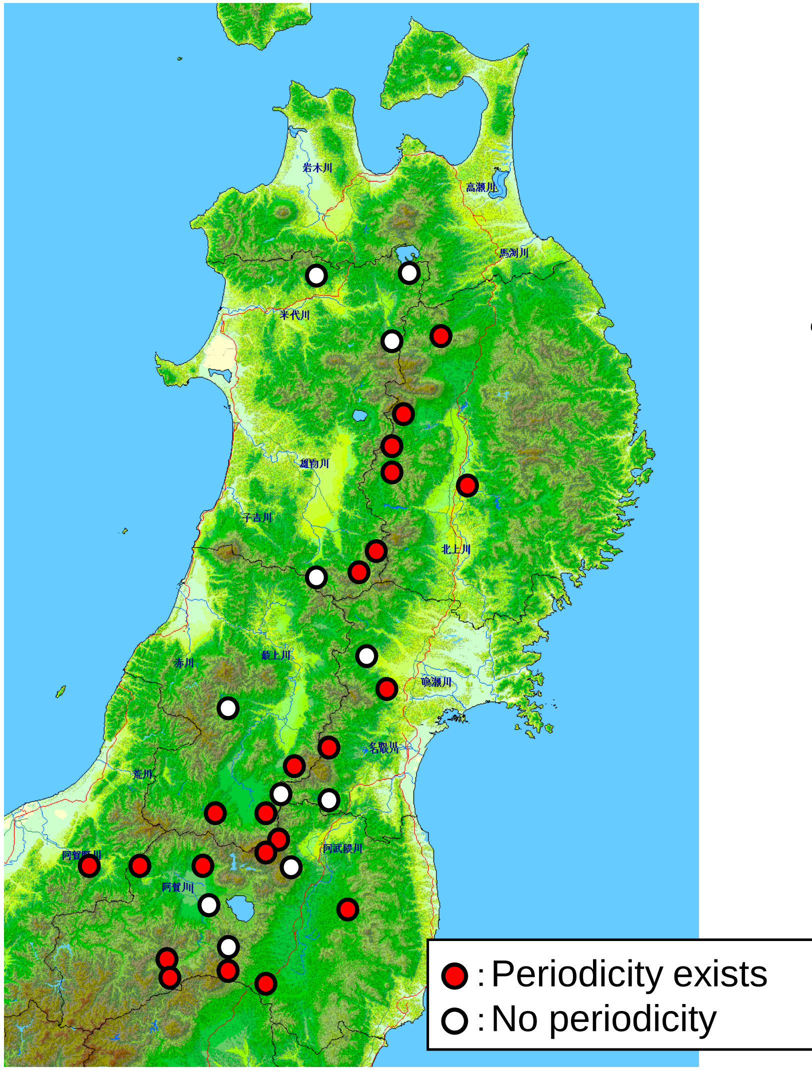




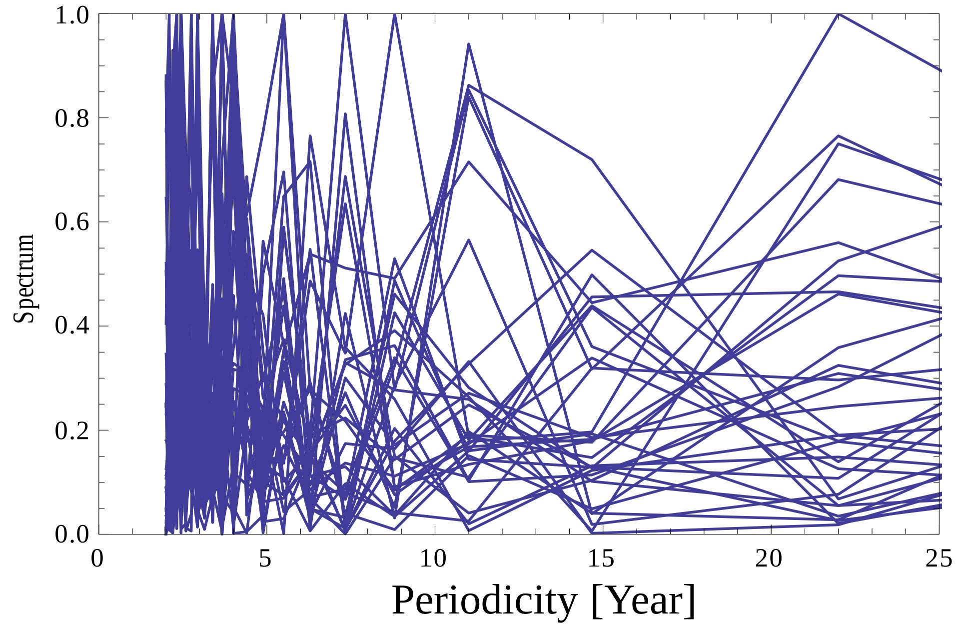
・In mountainous area of Hokkaido area, there is about **10 years periodicity** of annual maximum 3-days rainfall.

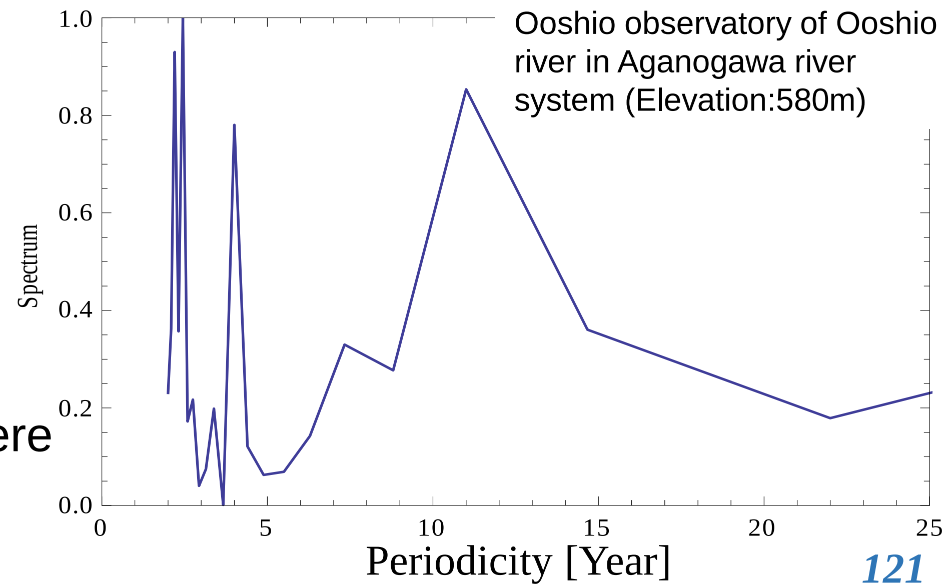
Periodicity of extreme rainfall in Tohoku area

Total number of data：44（1960～2003 [year]）



Spectrum of annual maximum 3-days rainfall

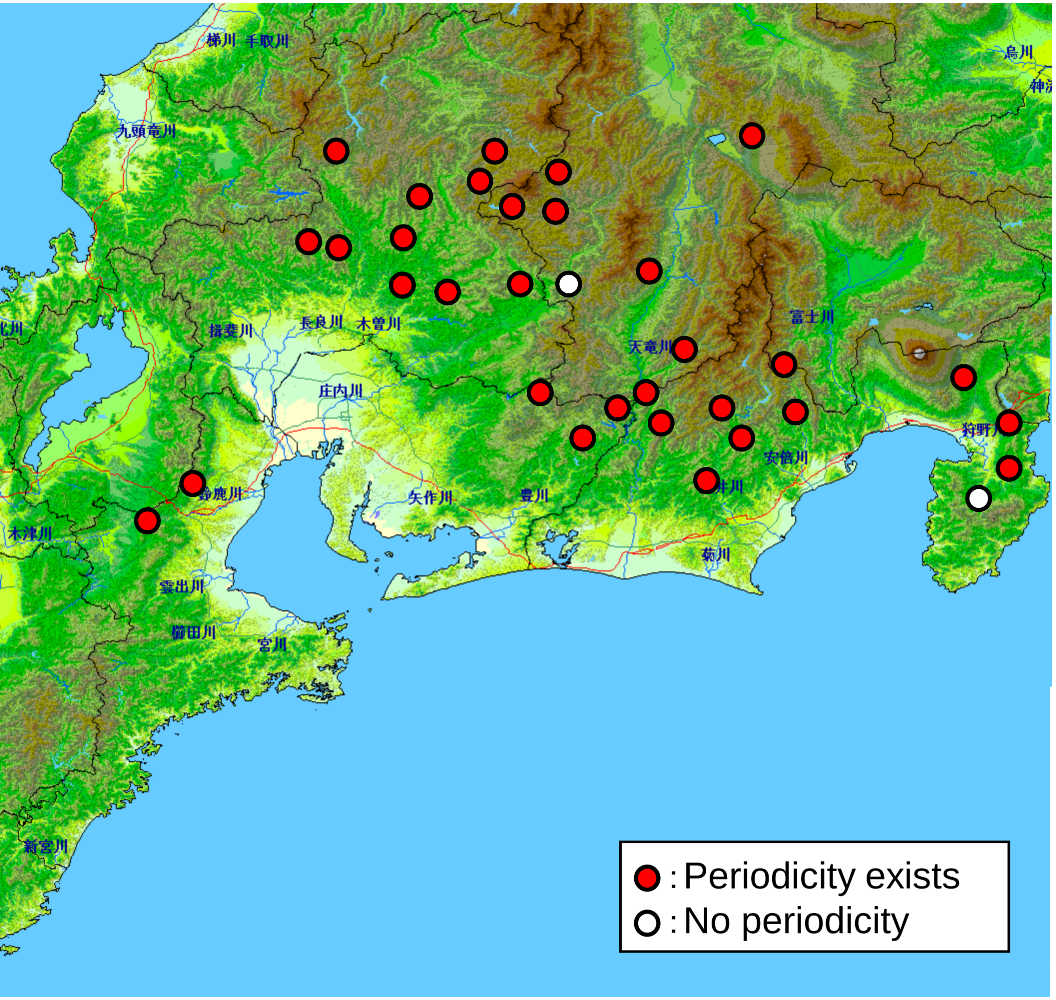




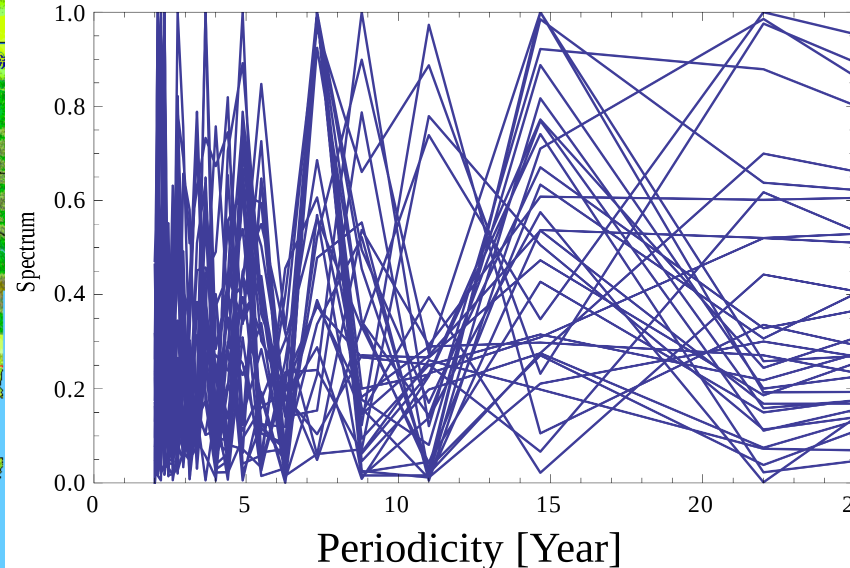
・In mountainous area of Tohoku area, there is about **10 years periodicity** of annual maximum 3-days rainfall.

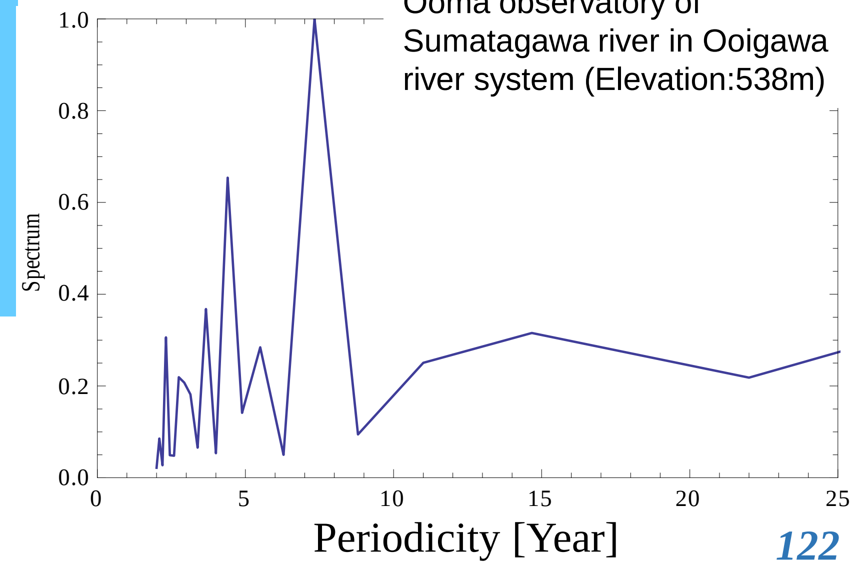
Periodicity of extreme rainfall in Chubu area

Total number of data：44（1960～2003 [year]）



Spectrum of annual maximum 3-days rainfall

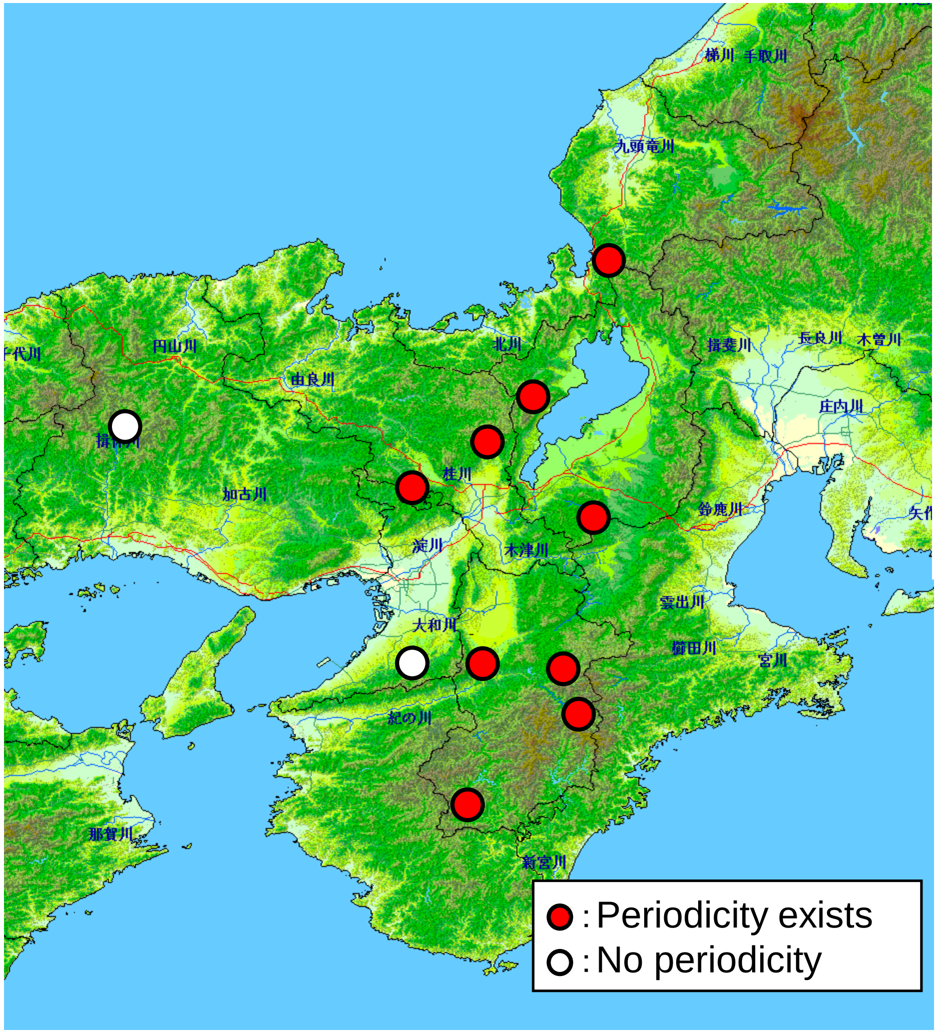




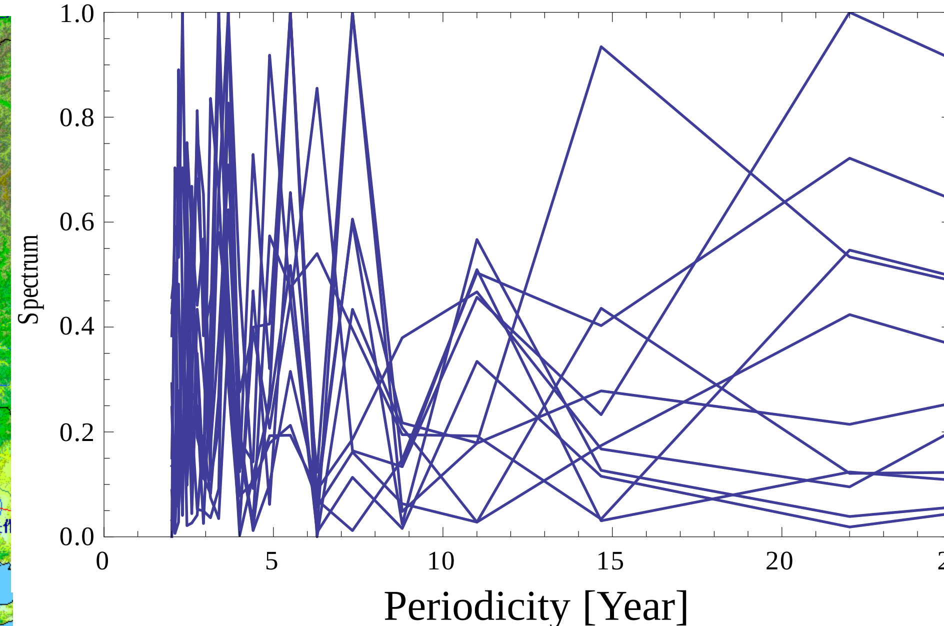
・In mountainous area of Chubu area, there is about **10 years periodicity** of annual maximum 3-days rainfall.

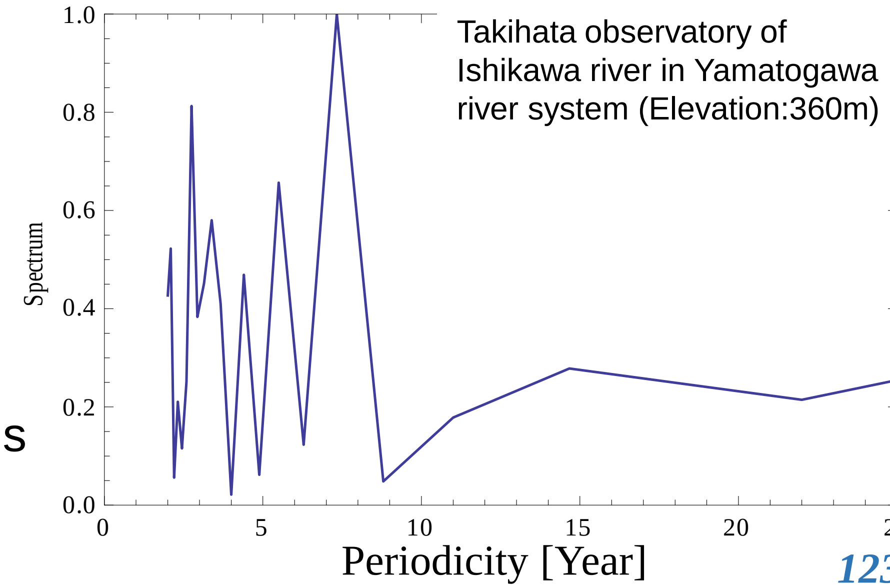
Periodicity of extreme rainfall in Kinki area

Total number of data：44（1960～2003 [year]）



Spectrum of annual maximum 3-days rainfall

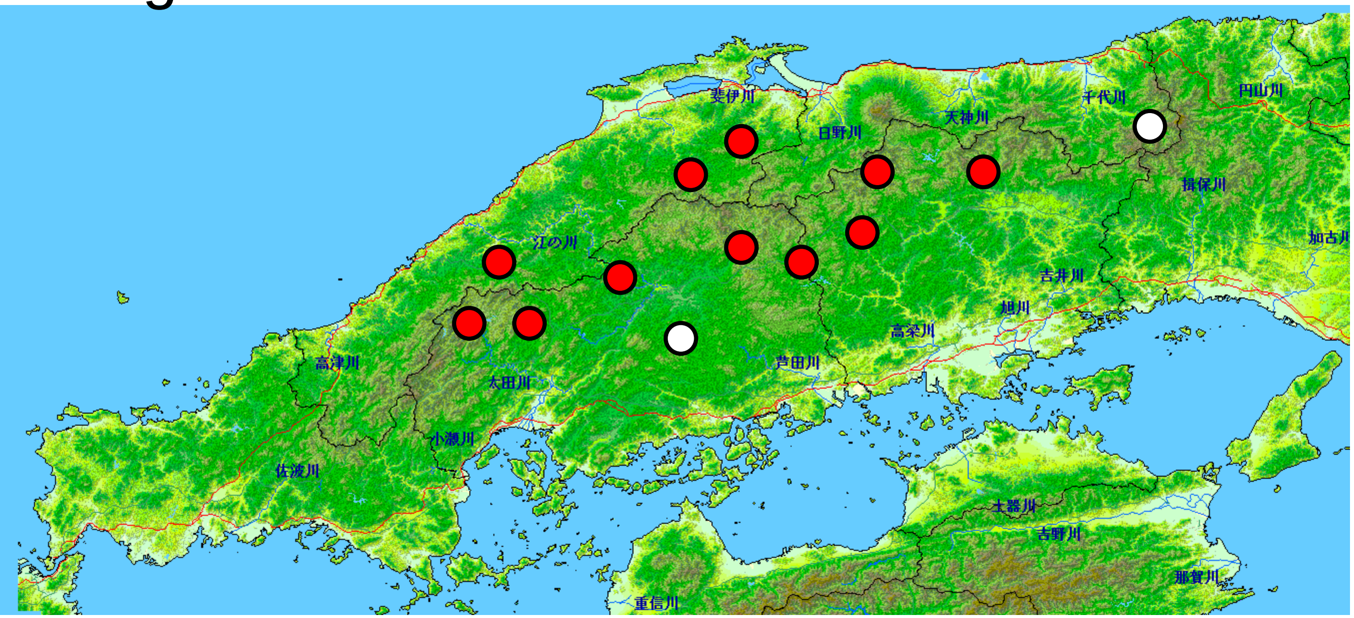




･In mountainous area of Kinki area, there is about **10 years periodicity** of annual maximum 3-days rainfall.

Periodicity of extreme rainfall in Chugoku area

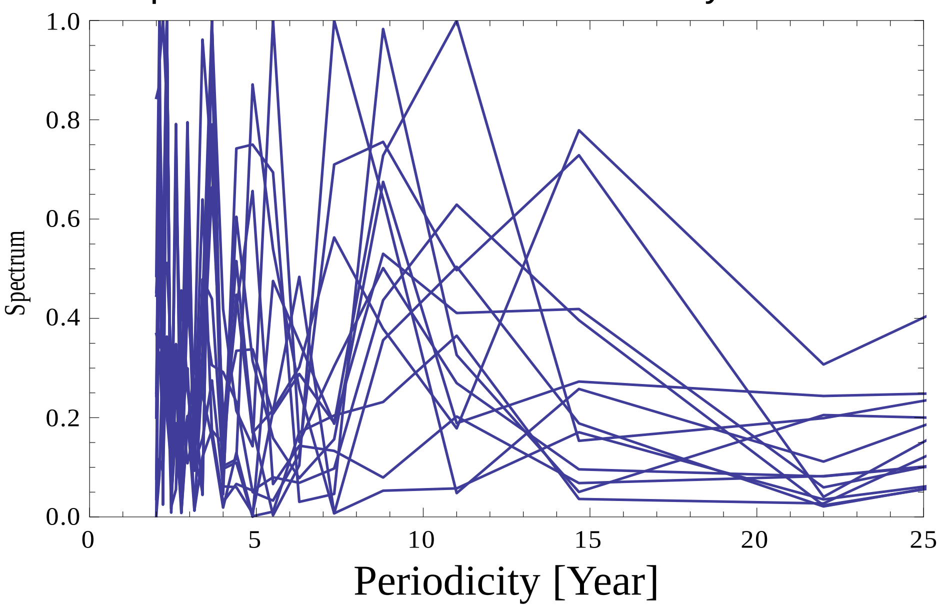
Total number of data：44（1960～2003 [year]）



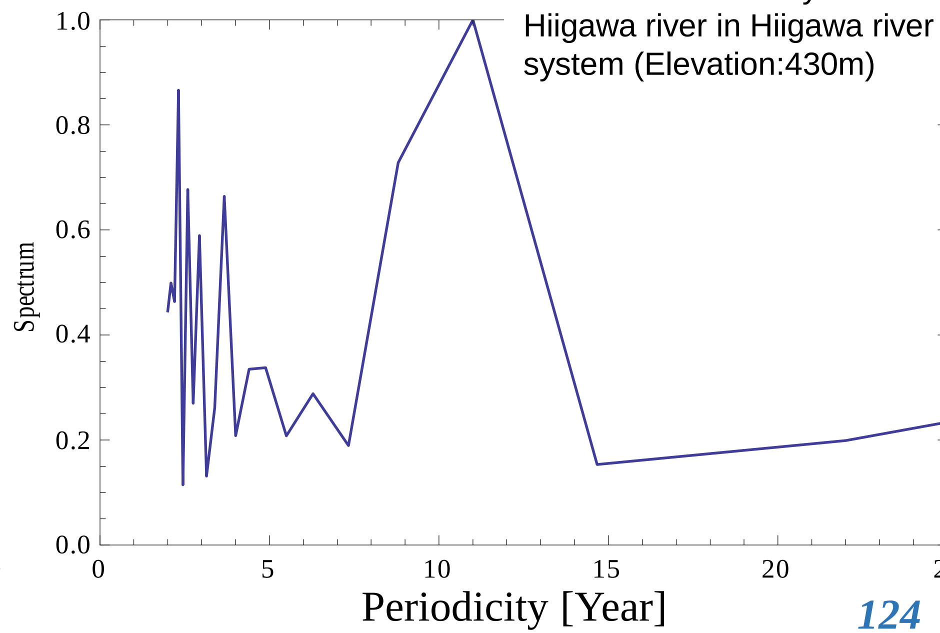
▪In mountainous area of Chugoku area, there is about **10 years periodicity** of annual maximum 3-days rainfall.

●：Periodicity exists
○：No periodicity

Spectrum of annual maximum 3-days rainfall

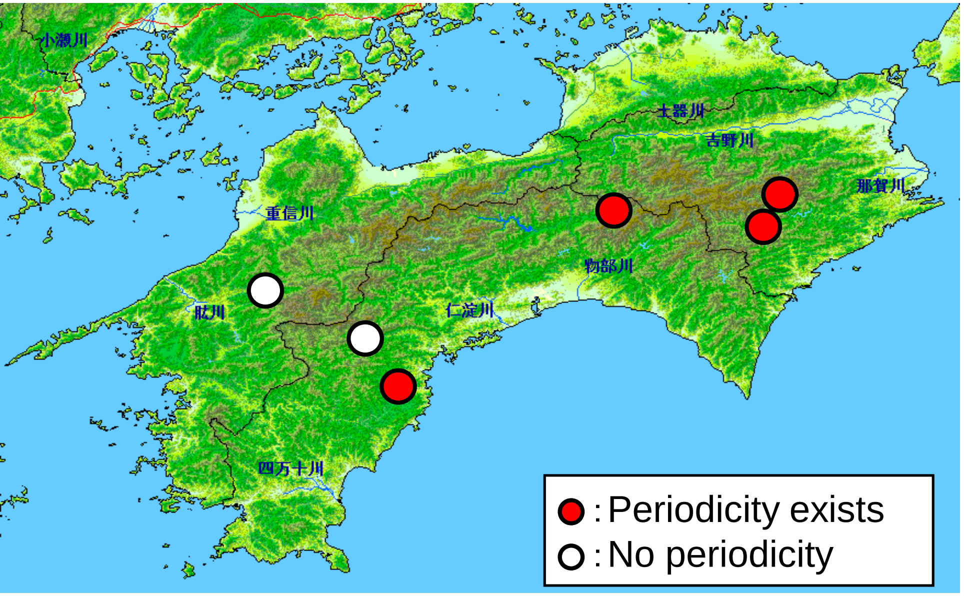


Torikami observatory of Hiigawa river in Hiigawa river system (Elevation:430m)

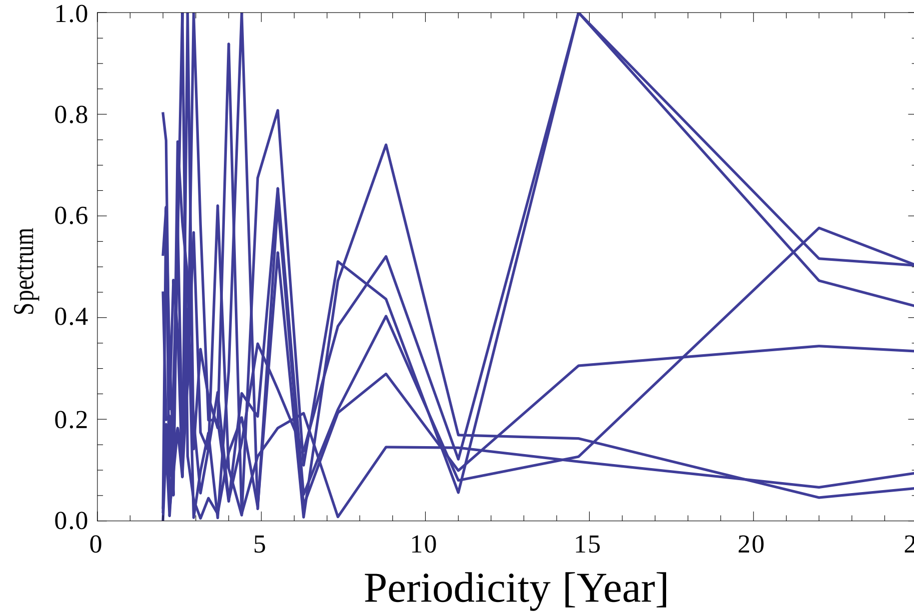


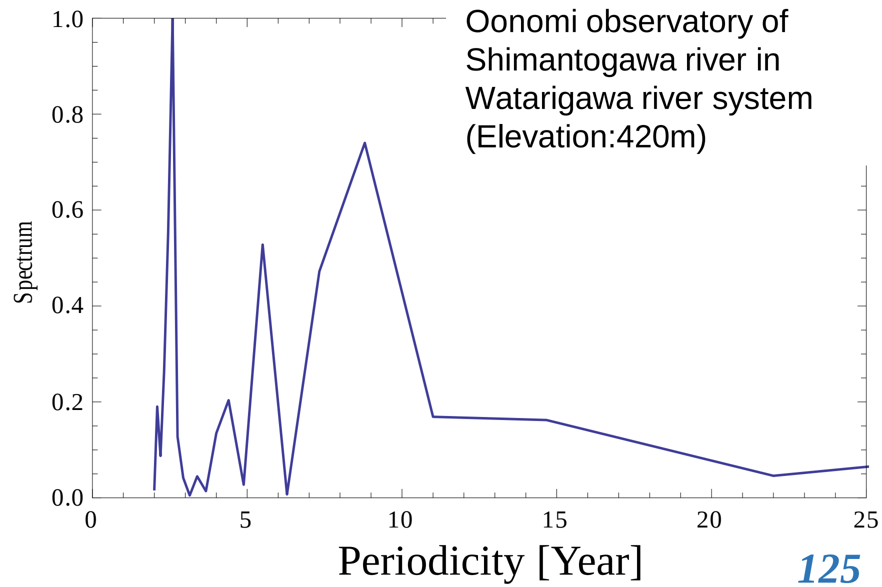
Periodicity of extreme rainfall in Shikoku area

Total number of data：44（1960～2003 [year]）



Spectrum of annual maximum 3-days rainfall



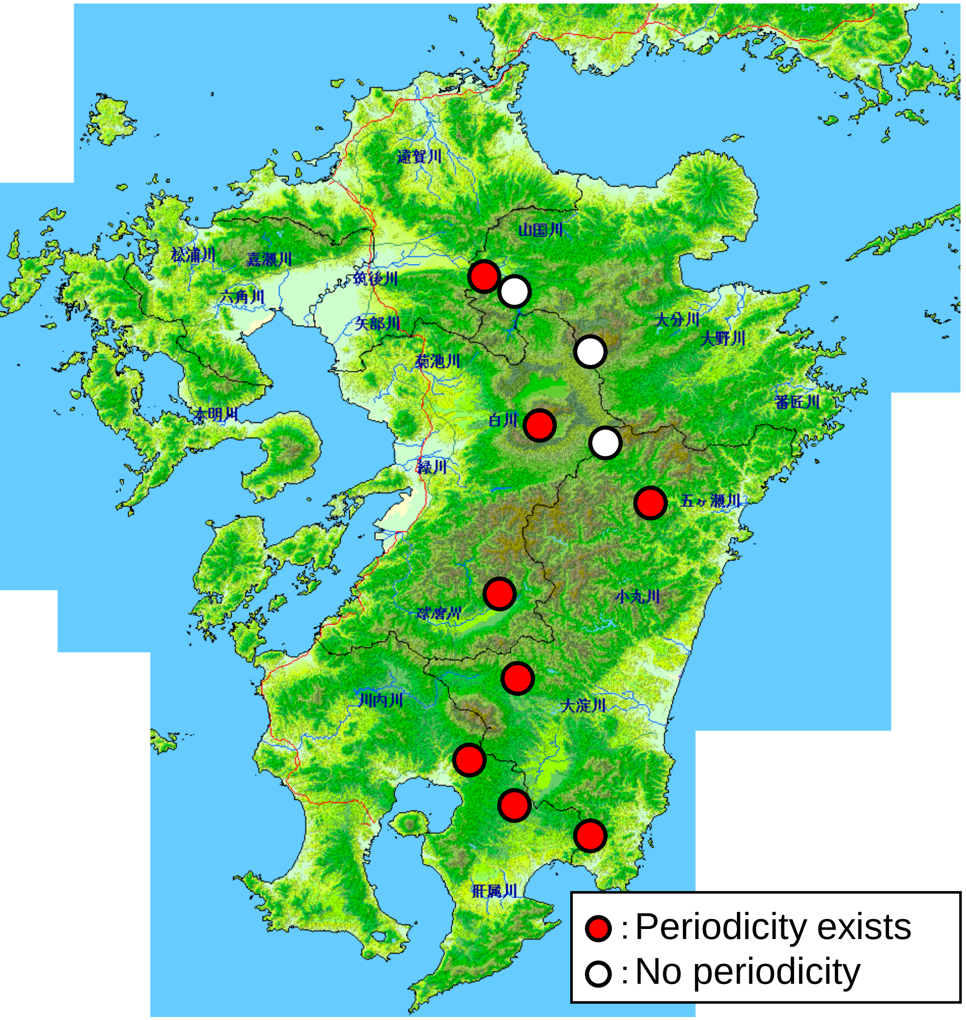


Oonomi observatory of Shimantogawa river in Watarigawa river system (Elevation:420m)

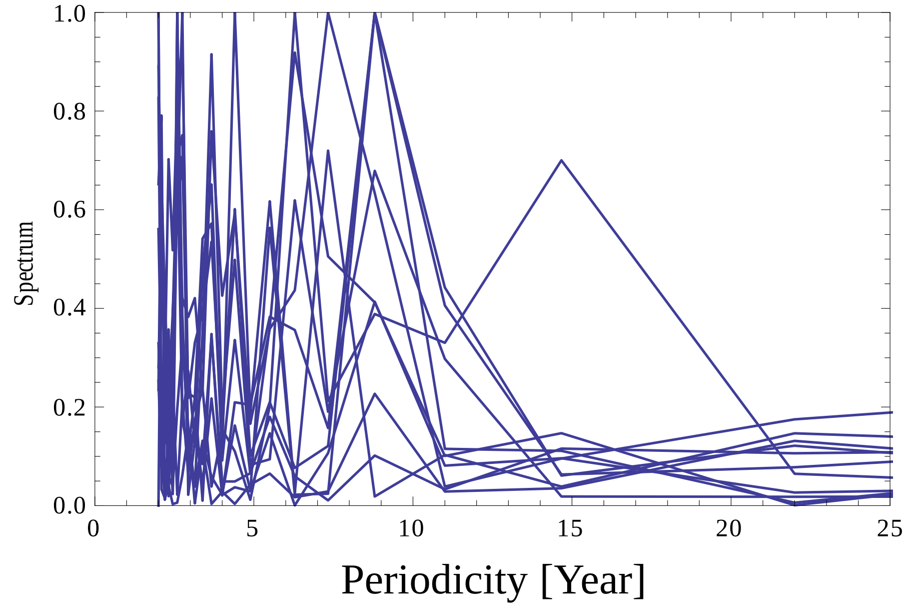
・In mountainous area of Shikoku area, there is about **10 years periodicity** of annual maximum 3-days rainfall.

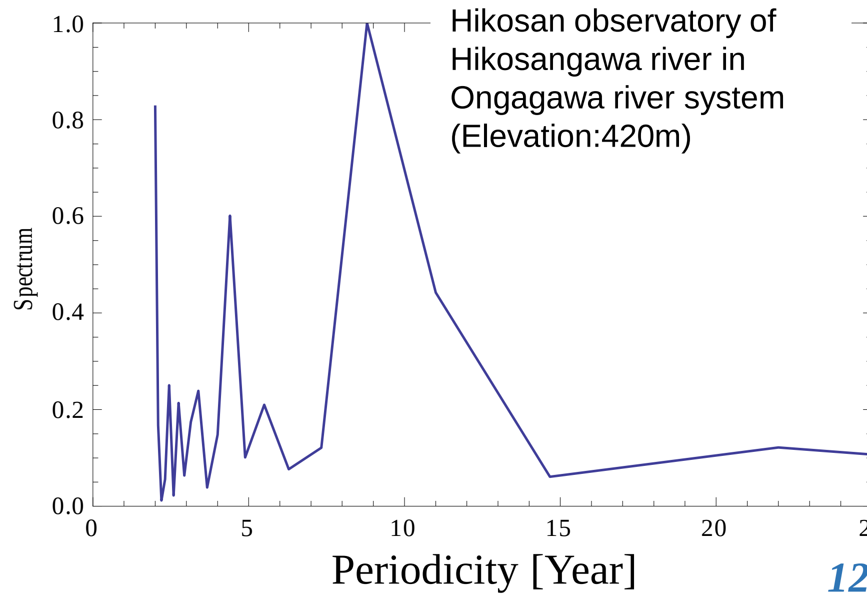
Periodicity of extreme rainfall in Kyushu area

Total number of data：44（1960～2003 [year]）



Spectrum of annual maximum 3-days rainfall

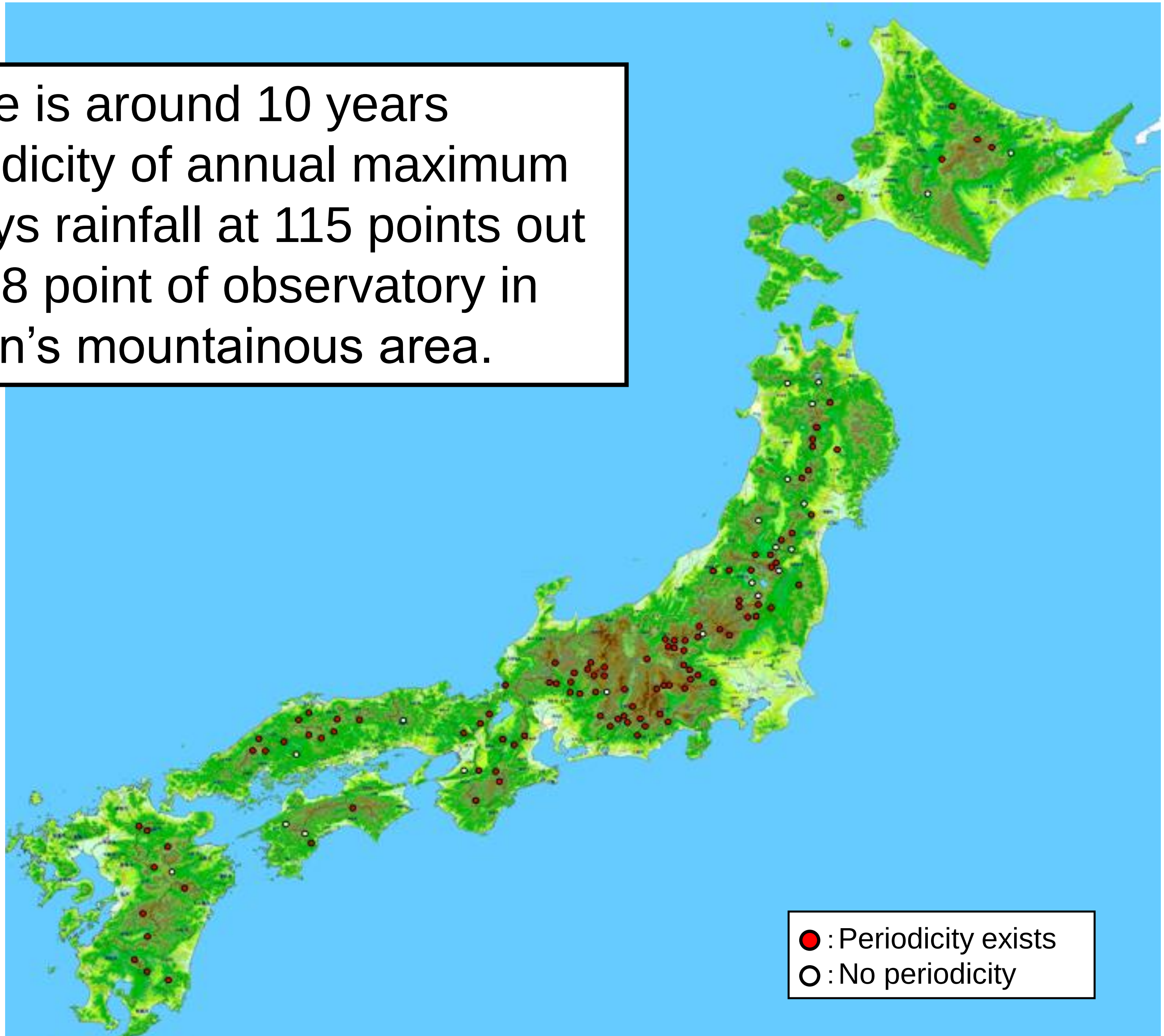




・In mountainous area of Kyushu area, there is about **10 years periodicity** of annual maximum 3-days rainfall.

Periodicity of extreme rainfall in Japan

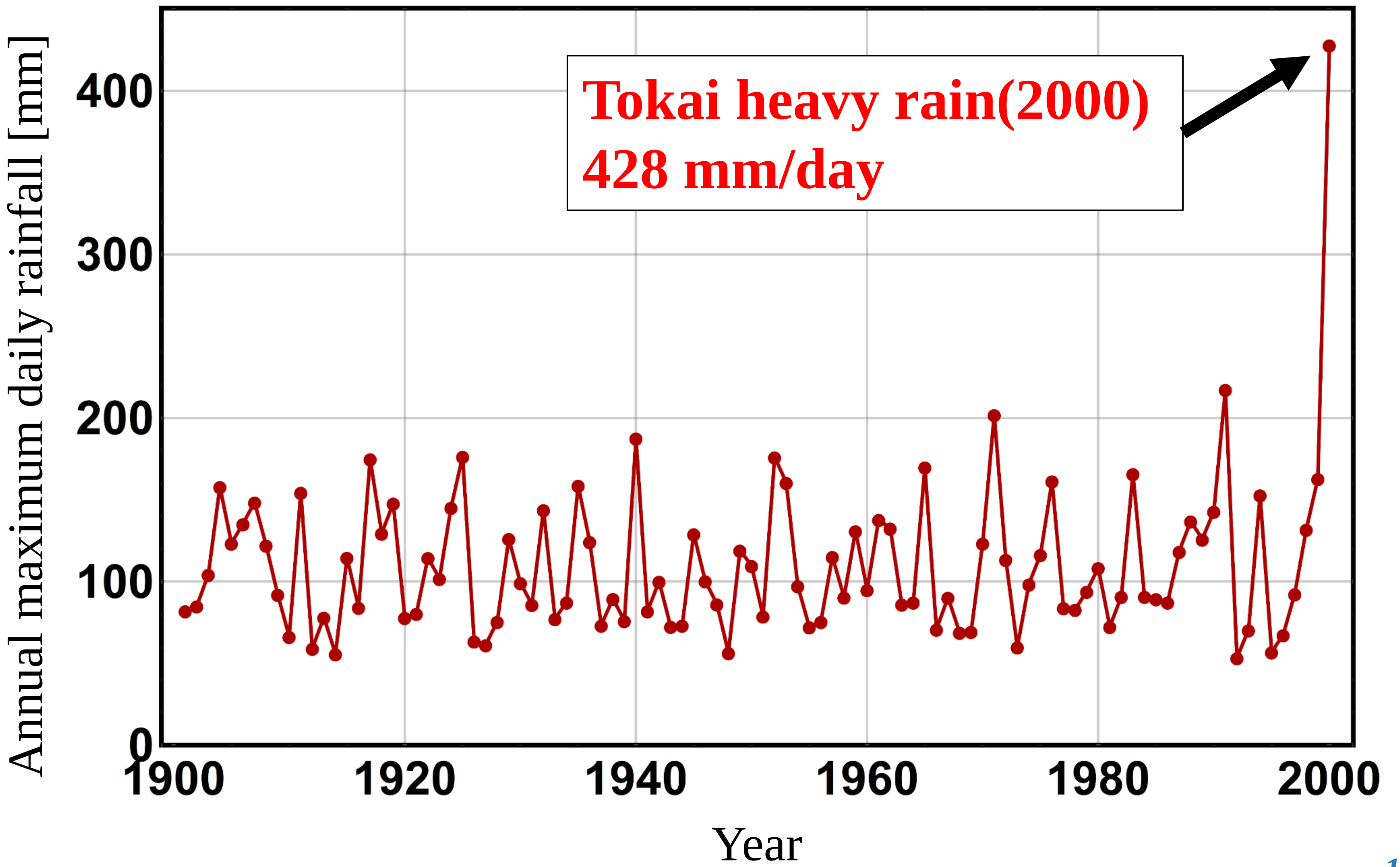
There is around 10 years periodicity of annual maximum 3-days rainfall at 115 points out of 138 point of observatory in Japan's mountainous area.



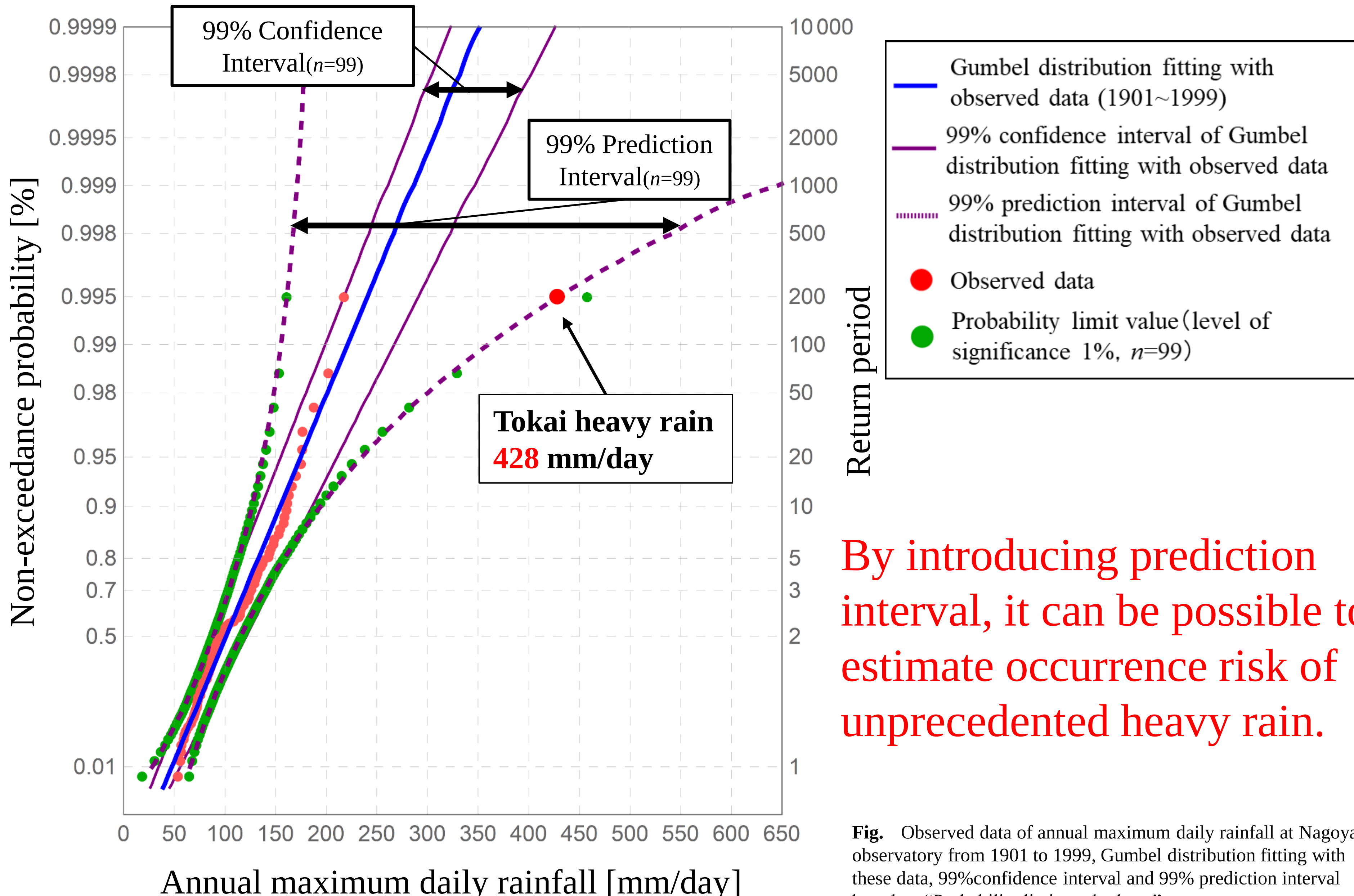
Frequency analysis of extreme hydrological quantity by using prediction interval

Is it possible to predict unprecedented heavy rain ?

By using observed data of annual maximum daily rainfall at Nagoya observatory from 1901 to 1999, we consider whether Tokai heavy rain can be predicted statistically.



Frequency analysis introducing prediction interval



By introducing prediction interval, it can be possible to estimate occurrence risk of unprecedented heavy rain.

Fig. Observed data of annual maximum daily rainfall at Nagoya observatory from 1901 to 1999, Gumbel distribution fitting with these data, 99%confidence interval and 99% prediction interval based on “Probability limit method test”.

Evaluation of heavy rainfall using prediction interval

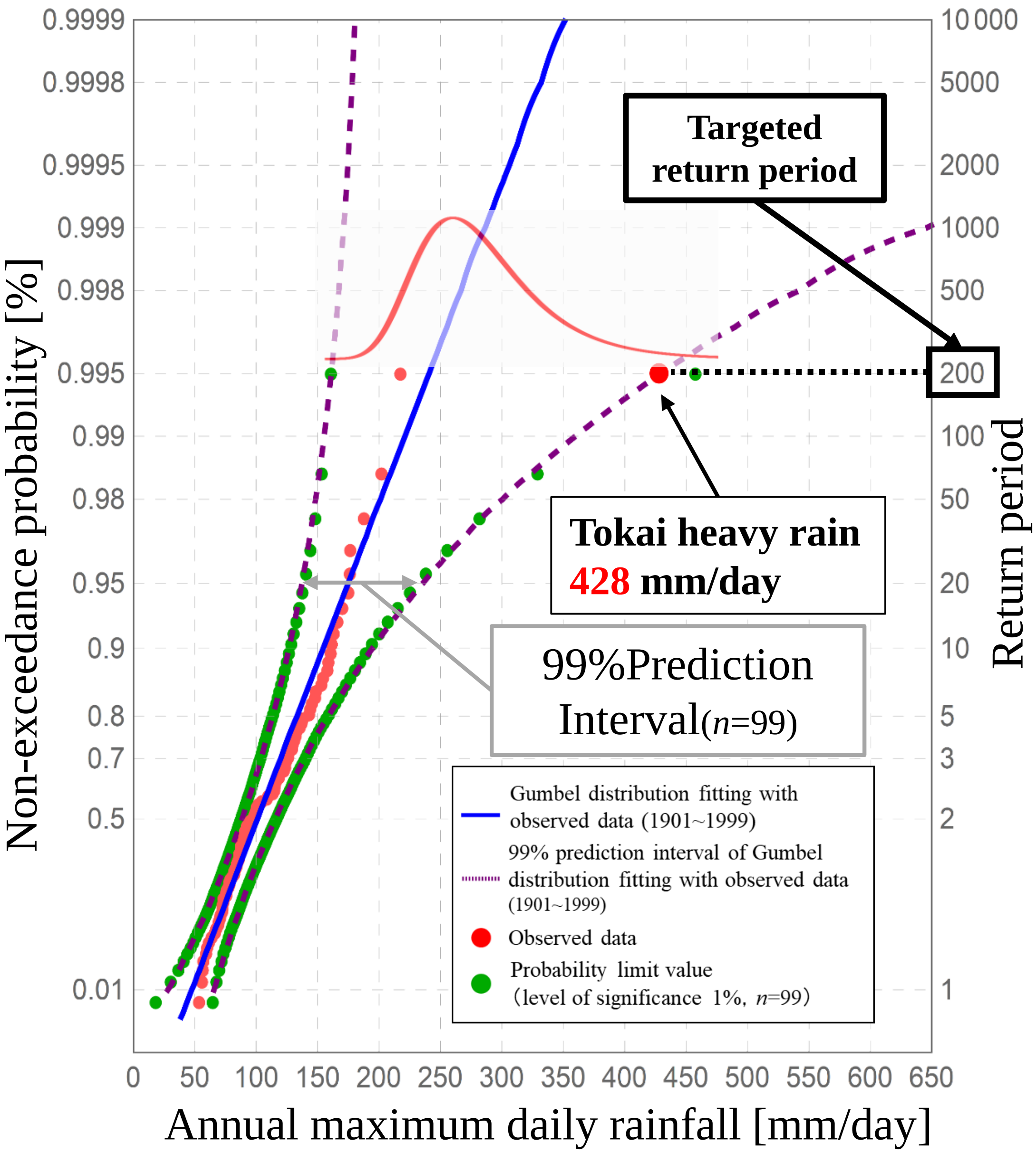


Fig. Observed data of annual maximum daily rainfall at Nagoya observatory from 1901 to 1999, Gumbel distribution fitting with these data, and 99% prediction interval based on "Probability limit method test".

Exceedance probability of prediction limit value is obtained by product of "targeted return period" and "exceedance probability of prediction interval".

Occurrence probability of "Tokai heavy rain"

$$\frac{1}{200} \times 0.005$$

Targeted return period

Exceedance prob. (99% P.I.)

$$= 2.5 \times 10^{-5} \quad (1/40000)$$

By introducing prediction interval, it can be possible to estimate occurrence risk of unprecedented heavy rain.

Relative evaluation of risk realized

[ref : **the rate of deaths**]

traffic accident : $1/(2\times10^4)$ [/year]

air plane accident : $1/(50\times10^4)$) [/year]

drug accident : $1/(200\times10^4)$ [/year]